Strategic Information Acquisition and Transmission*

Rossella Argenziano† Sergei Severinov‡ Francesco Squintani§

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Abstract

This paper explores the implications of costly information acquisition in the strategic communication models that follow Crawford and Sobel (1982). We revisit the fundamental insight that, because information is lost in communication, the decisions based on a biased expert's advice need to be imprecise. We surprisingly show that they may be more precise than optimal choices based on direct information acquisition, even if the expert is not more efficient than the decision maker at acquiring information. This result bears important implications for organization design, and especially for the study of the optimal authority allocation in the presence of incomplete information. Contrary to Ottaviani (2000) and Dessein (2002), we find that communication by an expert to the decision maker may often outperform delegation of the decision making authority to the expert. This result holds regardless of whether the expert is more or less efficient than the decision maker at acquiring information.

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†Department of Economics, University of Essex. rargenz@essex.ac.uk

‡Department of Economics, University of British Columbia, sseverinov@gmail.com

§Department of Economics, University of Warwick. f.squintani@warwick.ac.uk

1 Introduction

Strategic information transmission, also known as cheap talk, has been extensively analyzed in the economics literature under various assumptions on the underlying primitives, information and payoff structure.[1] Starting from the seminal contribution of Crawford and Sobel (1982), this literature has pointed out that the possibility of credible information transmission by an informed expert (sender) to an uninformed decision-maker (receiver), when the sender and the receiver disagree about the optimal decision, is limited: Some information will necessarily be lost in transmission. Put otherwise, the sender will transmit a noisy signal of her true information. Further, because information is lost in communication, the decisions based on a biased expert's advice need to be imprecise.

In turn, equilibrium loss of information in transmission implies that decision making based on the advice of a biased expert may not be an optimal organizational form. When an uninformed decision-marker can choose how to structure her interaction with an informed expert who has different interests, she will often find that the cheap talk outcome and the associated sender-receiver structure are suboptimal for her. Ideally, the decision-maker would prefer to have direct access to the information, rather than relying on an expert. If that is not possible, the decision maker would be better off if she delegated the decision-making authority to the informed expert, (Dessein (2002), Ottaviani (2000)), rather than retaining the decision-making authority while relying on the imperfect information communicated by the expert.

The above negative results about the possibility of credible information transmission in a cheap talk game are obtained for an exogenously given informational structure: It is typically assumed that the decision-maker is completely uninformed, and the sender is perfectly informed.[2] The latter had, presumably, acquired his information at no cost. This

[1]This literature includes Austen-Smith (1993), Gilligan and Krehbiel (1987, 1989), Krishna and Morgan (2001a, 2001b), Wolinsky (2002), Battaglini (2002, 2004), Ambrus and Takahashi (2008), and other works.

[2]Some exceptions are Austen-Smith (1994), Ottaviani (2000) and Ivanov (2010). In all these papers, the sender may or may not be informed. In Austen-Smith (1994), the sender may either acquire full information or remain completely ignorant. In the model by Ottaviani (2000), the amount of information available to the expert is exogenously given. In the model by Ivanov (2010), it can be selected costlessly by the decision-maker.

informational structure is clearly an extreme point in the set of feasible possibilities. Indeed, information is typically costly: Expertise and knowledge are obtained as a result of often time-consuming work and research. We investigate how the fact that information is typically costly and endogenously acquired affects the outcome of a cheap talk game, and identify a novel effect: The possibility of overinvestment in information acquisition by the sender. This result complements the result that information can be partially lost in transmission. We surprisingly show that the decisions based on a biased expert's advice may be more precise than optimal choices based on direct information acquisition, even if the expert is not more efficient than the decision maker at acquiring information.[3]

Also, the insight that the sender may overinvest in information acquisition allows us to revisit the analysis of the relative convenience, for a decision-maker, of different organizational forms such as the sender-receiver structure, delegation, and centralization. Contrary to Ottaviani (2000) and Dessein (2002), we find that communication by an expert to the decision maker may often outperform delegation of the decision making authority to the expert. This result holds regardless of whether the expert is more or less efficient than the decision maker at acquiring information.

Let us now briefly describe our set-up, the results and the intuitions behind them. We consider an environment in which both parties - the decision-maker and her agent - are initially uninformed about the state of the world and share a common prior. Information can be acquired by performing costly "experiments," or trials. The more trials are performed, the higher the precision of the information about the state of the world. In the limit, with infinitely many experiments, the state of the world is learned precisely and perfectly.[4]

We first analyze the sender-receiver structure: The agent acquires the information and then makes an announcement to the principal, who makes a decision. We consider both overt

[3] A recent paper by Che and Kartik (2009) studies a similar problem of information acquisition and communication. But while we consider cheap talk, their analysis is set in the opposite benchmark case of disclosure of verifiable information. Further, also the focus of the analysis is different. We compare the precision of decisions under communication and when the receiver acquires information directly. Che and Kartik (2009) do not consider this possibility. Instead, they show that divergence in prior beliefs between sender and receiver, while stifling communication, delivers better incentives for information acquisition.

[4] Out set-up is related to the Bernoulli-Uniform model of cheap talk analyzed by Morgan and Stocken (2008).

information acquisition (henceforth, the *overt game)*, where the number of trials performed by the expert is observed by the decision-maker, and the covert information acquisition case in which this number is only known to the expert –henceforth, the *covert game.* Our focus is on the amount of information *acquired and credibly transmitted* by the expert in equilibrium. In other words, on the quality of the final action made by the decision maker.

Next, we compare the outcome of both the overt and covert sender-receiver games to alternative organizational forms. In particular, we take the point of view of the decision maker and compare the convenience of relying on the advice of a biased expert to the possibility of either delegating both the information acquisition and the decision-making tasks to the agent (delegation), or retaining both tasks (centralization).

Our central results are the following. First, we provide sufficient conditions such that both the overt and the covert sender-receiver games have at least an equilibrium (the one preferred by the decision maker) in which the precision of the final decision is strictly larger than in the case of centralization. The expert acquires more information than what the decision-maker would if she had direct access to information. This more than offsets the tendency to lose some of it in transmission. These conditions are satisfied for moderate values of the bias and of the information acquisition cost. Second, we find weaker conditions guaranteeing that in all the Pareto-undominated equilibria of both the overt and the covert game, the precision of the final decision is at least as large as in centralization.

These results have immediate implications for the comparison of organizational structures. Suppose that the decision maker can obtain for free (as the cost is borne by the expert) an amount of information at least as large as what she would optimally choose to acquire if she acquired it personally and bore the cost. Then, she will clearly prefer the sender-receiver structure to centralization. Similarly, she will prefer the sender-receiver structure to full delegation: The precision of the final decision is weakly higher in the former case, and the decision is biased by the expert's preferences in the latter case. These results stand in contrast with the findings of Ottaviani (2000) and Dessein (2002) that delegation usually outperforms communication, when the amount of information held by the expert sender is exogenously given. Instead, in our model, communication often outperfoms delegation, and this result is independent of the information acquisition cost of the decision maker.[5]

Let us now illustrate the two key intuitions that drive our results. The first strategic effect that our setup allows us to highlight is the possibility of overinvestment in information by the sender. This is based on the fact that the presence of strategic considerations makes the value of information for the expert in a communication game different from the value of information for a decision maker who directly acquires information. The mechanism through which this leads to overinvestment is different in the overt and the covert game.

Consider the overt game first. The cheap talk stage that follows information acquisition is characterized by a multiplicity of equilibria endemic to most cheap talk games. In particular, there is always a "babbling equilibrium" in which the principal completely ignores the signal sent by the agent. This multiplicity of equilibria provides the principal with an instrument to induce the agent to acquire more information than the latter would like. Precisely, the principal could and would credibly threaten the agent to play a "babbling" equilibrium unless the agent performs a certain number of trials. An expert's advice would only be taken into account if the expert had invested a large amount in information acquisition. Since the principal does not bear the cost of information acquisition, she would be disposed in favor of a larger number of experiments. Hence, this factor works in favor of overinvestment.

We find that the latter factor (overinvestment) dominates the loss of information in transmission for moderate values of the bias and of the investment cost. As a result, the decision-maker's information and, hence her action, would be more precise than under centralization (or delegation). As a consequence, in those cases the decision-maker would prefer the sender-receiver structure to the alternative organizational forms.

[5] In a recent paper, Thordal-Le Quement (2010) analyzes a model in which the decision maker can choose between retaining and delegating the decision power *after* observing the amount of information acquired by the expert. He shows that this provides the expert with an incentive to acquire a large amount of information in order to guarantee that the decision power will be delegated to him.

When information acquisition is covert instead, deviations are not observable, hence the threat of a babbling equilibrium in case of underinvestment is no longer available. However, overinvestment in information may still arise due to a different and novel factor appearing in our model. This factor stems from the inflexibility of the equilibrium language when information acquisition is covert. Indeed, note that the equilibrium language i.e., the mapping between the set of feasible messages and the choices of the receiver, is determined by the receiver's equilibrium expectations about the number of experiments to be performed by the sender. Therefore, it is not sensitive to a sender's deviation to a different number of experiments. This property limits the attractiveness of a deviation to a non-equilibrium number of signals for the sender: Regardless of the number of trials that she conducts, any message she sends will be interpreted under the belief that she acquired the equilibrium (rather than the actual) number of signals.[6,7]

The second strategic effect that our setup allows us to highlight arises from the observation that, when information is costly, the sender may be reluctant to acquire more information than she would transmit in the equilibrium. As pointed out by Ivanov (2010) and Fischer and Stocken (2001), a biased expert with a limited amount of information has a higher incentive than a fully informed expert to reveal it truthfully to a decision maker. In our setup, information is costly acquired, hence it does not make sense for the sender to pay for pieces of information that would be at least partially lost in transmission. This cost consideration creates a strong force against the loss of information in transmission. As a result, it is possible in our model that there is very little, or even no information loss in the equilibrium of the communication games.

Our weaker, but more general result, is based on the latter strategic effect. We show that under some conditions, the precision of the final decision is at least as large as in centralization in all Pareto undominated equilibria of both the overt and covert game. In

[6] The latter intuition is related to the analysis in Eso and Szalay (2011) of the incentives of an uninformed sender to acquire full information about the state of the world in a cheap talk game. In their paper, information acquisition is covert, and it is a binary choice: The sender can either stay uninformed, or learn the state of the world at a cost.

[7] In Section 4.4 we discuss the robustness of our results in a setting where the expert can also send a message to the decision maker about the number of trials performed.

particular, this result applies to the equilibrium of the overt game in which the most informative equilibrium is selected in every communication subgame, on and off the equilibrium path.

Our stronger result instead, namely the existence of equilibria of the overt and covert game that involve decisions strictly more precise than under centralization, is based on the combination of both forces: the possibility of overinvestment, and the cost considerations that act agains information loss in transmission. For both results, we provide sufficient conditions that involve full revelation in equilibrium. This simplifies the analysis, but full revelation is by no means a necessary condition, as we show numerically.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 discusses the characterization of the equilibria of the covert and overt sender-receiver games. Section 4 focuses on the precision of the decision in the equilibrium outcomes of both games, and its comparison with the precision of the decision in the centralized outcome. Section 5 derives the implications for organization design. Section 6 concludes.

2 The Model

We start by introducing our model of cheap talk with endogenous acquisition of costly information by the sender. It is a natural extension of the classic Crawford and Sobel (1982) uniform-quadratic model. There are two players, the expert and the decision maker. At the end of the game, the decision maker chooses an action $y \in [0, 1]$ to maximize her payoff

$$U^R(y, \theta) = -(y - \theta)^2, \tag{1}$$

where θ is an unknown state of the world, with uniform common prior distribution on $[0, 1]$. At the beginning of the game, the expert can purchase some information on θ. Specifically, he can acquire $n \in \mathbb{N}$ i.i.d. binary trials, with probability of success equal to θ, at a linear cost $c(n) = cn$. If information acquisition is overt, n is observed by the decision maker and we refer to this case as the *overt game*. If information acquisition is covert, n is private, unverifiable information of the expert and we refer to this case as the *covert game*. The number of trials n measures the precision of the expert's information about θ. We let k be the number of successes, and note that it is distributed according to the binomial distribution:

$$f(k; n, \theta) = \frac{n!}{k!(n-k)!}\theta^k (1-\theta)^{n-k}, \text{ for } 0 \leq k \leq n.$$

For future reference, we also note that the unconditional distribution of k is uniform:

$$\Pr(k; n) = \int_0^1 \frac{n!}{k!(n-k)!}\theta^k (1-\theta)^{n-k} d\theta = \frac{1}{n+1}.$$

Before the decision maker acts, the expert may communicate a message $m \in [0, 1]$ to the decision maker. The players' preferences are misaligned: The expert's payoff is:

$$U^S(y, \theta, b) - c(n) = -(y - \theta - b)^2 - cn, \tag{2}$$

where the bias b measures the preference discrepancy among players.

We note that the posterior distribution of θ after observing k successes in n trials is a Beta distribution with parameters $k+1$ and $n-k+1$:

$$f(\theta; k, n) = \frac{(n+1)!}{k!(n-k)!}\theta^k (1-\theta)^{n-k}, \text{ if } 0 \leq \theta \leq 1.$$

The corresponding posterior expectation is $E[\theta | k, n] = \dfrac{k+1}{n+2}$.

3 The Equilibrium Characterization

3.1 The Overt Game Equilibria

A pure strategy Perfect Bayesian equilibrium of the overt game is described by $\left(n, \{P_{n'}\}_{n' \geq 0}, \{\mathbf{y}(P_{n'})\}_{n' \geq 0}\right)$. The integer n represents the number of trials conducted by the expert in equilibrium. For any number of trials n', the partition $P_{n'}$ of the set of expert's types $\{0, 1, ..., n'\}$ describes the information communicated if n' trials were conducted. For any element, or pool, p of the communication partition $P_{n'}$, the expert's message m informs the decision maker only that the expert's type k belongs to p. In a babbling communication partition, $P_{n'}$ has a single element, whereas in a fully separating communication partition, each element of $P_{n'}$ contains a single type. For any communication partition $P_{n'}$ and any pool p in $P_{n'}$, the action $y_p\left(P_{n'}\right) \in [0,1]$ describes the decision maker's decision upon being informed that $k \in p$.

By backward induction, we solve the game starting from the communication subgames that follow the information acquisition stage. By sequential rationality, the decision maker chooses y to maximize her payoff given the expert's message m and the equilibrium beliefs about θ. Due to the payoff's quadratic loss specification, she chooses y to match the expectation of θ given her information. Specifically, we show in the Appendix that for every n', $P_{n'}$ and $p \in P_{n'}$, upon being informed that k belongs to p, the decision maker chooses

$$y_p\left(P_{n'}\right) = E\left[\theta|p\right] = \frac{1}{|p|}\sum_{k\in p}\frac{k+1}{n'+2}, \tag{3}$$

where $|p|$ denotes the cardinality of p.[8]

Given a sequentially rational list of actions $\mathbf{y}\left(P_{n'}\right)$, the sender's communication strategy, described by the partition $P_{n'}$, must be incentive compatible in equilibrium. Specifically, for any $p \in P_{n'}$, and $k \in p$, incentive compatibility requires that:

$$\int_0^1 U^S\left(y_p\left(P_{n'}\right), \theta, b\right) f\left(\theta; k, n'\right) d\theta \geq \int_0^1 U^S(y_q\left(P_{n'}\right), \theta, b) f\left(\theta; k, n'\right) d\theta, \text{ for all } q \in P_{n'}. \tag{4}$$

In the uniform-quadratic model by Crawford and Sobel (1982), the incentive-compatible communication strategies are characterized as partitions of the type space where each element i is an interval (a_i, a_{i+1}), and where each marginal type a_i is exactly indifferent between inducing the two sequentially-rational actions y_i and y_{i+1} associated with the intervals (a_{i-1}, a_i) and (a_i, a_{i+1}). This characterization implies the so-called "arbitrage condition",

[8]Indeed, one remarkable feature of our statistical model is that the expectation of θ conditional on the pool p equals the average expectation of θ conditional on k, across the types $k \in p$.

$a_{i+1} - a_i = a_i - a_{i-1} + 4b$, which pins down all equilibria.

The characterization of incentive-compatible partitions $P_{n'}$ in our model is analogous to the characterization by Crawford and Sobel (1982). Specifically, we prove in the Appendix that each pool p in an equilibrium partition $P_{n'}$ is composed of consecutive types. Further, the lowest type in a pool p_i must prefer the associated sequentially rational y_i to the action y_{i-1} associated to the immediately lower element p_{i-1}. Likewise, the highest type in p_i must prefer y_i to the action y_{i+1} associated to the immediately higher pool p_{i+1}. These results lead to the following characterization.

Proposition 1 *For any n', any incentive compatible communication partition $P_{n'}$ composed of I pools is such that each pool $p \in P_{n'}$ is a set of consecutive types, and the cardinalities $|p_i|$ and $|p_{i+1}|$ of p_i and p_{i+1} satisfy the following condition for any $i = 1, ..., I-1$:*

$$4b\left(n'+2\right) - 2 \leq |p_{i+1}| - |p_i| \leq 4b\left(n'+2\right) + 2. \tag{5}$$

As we show in the Appendix, Proposition 1 has the following implications. First, a fully separating partition is incentive compatible if and only if $b\,(n'+2) \leq 1/2$. Second, if $b \geq 0.25$, the only incentive compatible communication partition is a babbling one.

Notice that condition (5) is conceptually equivalent to the arbitrage condition by Crawford and Sobel (1982). This fact underlines the direct comparability of our results with that canonical construction. The only difference is that, in our model, there are no exactly indifferent marginal types, as the type space is finite. Furthermore, as $n' \to \infty$, any incentive compatible partition P of our model converges to an equilibrium partition of the Crawford and Sobel (1982) model in which the expert is perfectly informed. Precisely, $|p_i|/(n'+1) \to a_i - a_{i-1}$ for any i.

Before turning to describe the information acquisition choice, we note that given the quadratic loss function specification, the equilibria of the communication subgames can be Pareto ranked on the basis of the associated expected residual variance. For any equilibrium partition $P_{n'}$, in the communication subgames the expert's ex-ante utility $E\left[-\left(y_p-\theta-b\right)^2|P_{n'}\right]$ and the decision maker's ex-ante utility $E\left[-\left(y_p-\theta\right)^2|P_{n'}\right]$ differ only by a constant:[9]

$$E\left[-\left(y_p-\theta-b\right)^2|P_{n'}\right]=E\left[-\left(y_p-\theta\right)^2|P_{n'}\right]-b^2. \tag{6}$$

Hence, any Pareto-efficient equilibrium communication partition $P_{n'}$ must minimize the expected residual variance of θ, $E\left[\left(y_p-\theta\right)^2|P_{n'}\right]$.

We prove in the Appendix that for given number of trials, the equilibrium partition that minimizes the expected residual variance is the one with the highest number of elements and the smallest heterogeneity in the cardinalities of the elements. An immediate implication is that if full separation is an equilibrium of a communication subgame, it is Pareto efficient.[10]

To conclude the characterization of the equilibria of the overt game, we observe that given a collection $\{P_{n'}\}_{n'\geq 0}$ of incentive compatible partitions that satisfy condition (5), and associated sequentially rational vectors of actions $\{\mathbf{y}\left(P_{n'}\right)\}_{n'\geq 0}$, the equilibrium number of trials n must maximize the ex-ante expert's utility:

$$\sum_{k=0}^{n'}\int_0^1 U^S\left(y^{k,n'},\theta,b\right)f\left(\theta;k,n'\right)d\theta\Pr\left(k;n'\right)-c(n'), \tag{7}$$

where $y^{k,n'}$ denotes the action $y_p\left(P_{n'}\right)$ such that $k\in p$.

The game may have many equilibria, because of the equilibrium multiplicity in the communication subgames. We focus the analysis on Pareto undominated equilibria. This requirement implies that on equilibrium path, that is in the subgame that follows the acquisition of the equilibrium number of trials n, players play the communication equilibrium which minimizes the expected residual variance. But it is possible that players coordinate on sub-optimal equilibria off the equilibrium path, i.e. for communication subgames that

[9] Notice that in the communication stage, the information acquisition cost $c(n)$ is sunk.

[10] In fact, full separation corresponds to the partition $P_n=\{\{0\},\{1\},...,\{n\}\}$ composed of singletons, and for any pool p in any partition P_n, $E[\theta|p]=\sum_{k\in p}E[\theta|k]/|p|$. Hence, the result follows by the Jensen inequality, because the square is a convex function.

follow the acquisition of n' trials, for any $n' \neq n$.

3.2 The Covert Game Equilibria

We now turn to describe the covert game equilibria. A pure-strategy Perfect Bayesian Equilibrium of the covert game is described by a triple $(n, P_n, \mathbf{y}(P_n))$. The equilibrium number of trials n maximizes the ex-ante expert's utility. The communication partition P_n of the set of expert's types $\{0, 1, ..., n\}$ must be incentive compatible given $\mathbf{y}(P_n)$, and the associated vector of decision maker's actions $\mathbf{y}(P_n)$ must be sequentially rational.

Unlike in the overt game, when choosing the final action the decision maker cannot detect any expert's deviation at the information acquisition stage. Hence, she must maintain her equilibrium belief that the expert acquired n trials, and that he communicated according to the partition P_n. As a result, even if the expert chooses a number of trials n' other than n, his subsequent messages can only induce one of the actions in the equilibrium list $\mathbf{y}(P_n)$. In the most profitable deviation, each type $k \in \{0, 1, ..., n'\}$ chooses optimally which action to induce given his beliefs on θ. So, a triple $(n, P_n, \mathbf{y}(P_n))$, constitutes an equilibrium only if the number of trials n maximizes the following expression:

$$\sum_{k=0}^{n'} \left[\max_{y_p \in \mathbf{y}(P_n)} \int_0^1 U^S(y_p, \theta, b) f(\theta; k, n') \, d\theta \right] \Pr(k; n') - c(n'). \tag{8}$$

As for the overt game, we focus the analysis on Pareto undominated equilibria. But unlike in that case, we cannot rely on the result that the equilibrium partition P_n of any Pareto undominated equilibrium with n trials is the most informative, that is the one that minimizes $E\left[(y_p - \theta)^2 | P_n\right]$. It is possible, in principle, that in the candidate equilibrium with n trials and the most informative communication partition there is a profitable deviation for the expert at the information acquisition stage, while in a candidate equilibrium with n trials and a less informative communication partition there are no profitable deviations.

4 Decision Precision

4.1 Direct Information Acquisition

The fundamental insight of standard communication models á la Crawford and Sobel (1982) is that decisions based on information communicated by a biased expert are less efficient than those that would be made if the decision maker had direct access to the information. We revisit this issue in the context of our model, in which information is costly and is endogenously acquired. As in the uniform-quadratic case in Crawford and Sobel (1982), we measure the information associated to an equilibrium as minus the residual variance of the equilibrium action $E\left[-(y_p-\theta)^2 |P_n\right]$. Surprisingly, we find that the decisions based on the advice of a biased expert can be *more precise* than the decisions based on information directly acquired by the decision maker, even assuming that the cost of information acquisition is the same across players.

This result follows from two intuitive observations. First, when information is costly and endogenously acquired, the fact that the cost is borne by the expert creates an externality not internalized by the decision maker. This can generate overinvestment in information. Second, when information is costly and endogenously acquired, the expert is likely to be only imperfectly informed. This, in turn, mitigates the problem of information loss in the communication between the expert and the decision maker. The amount of information transmitted by an equilibrium partition is decreasing in the heterogeneity of the elements of the partition. An implication of the incentive compatibility condition (5) is that the minimum heterogeneity required to guarantee incentive compatibility is increasing in the amount of trials performed by the expert. The combination of these two factors, namely overinvestment and a small loss in information in transmission, drives the result.

The possibility of overinvestment in a Pareto-undominated equilibrium of the overt game is intuitive in the decision maker's preferred equilibrium. The latter is constructed by choosing the number of trials n and communication partition P_n on path that maximize the decision maker's ex-ante utility $E\left[-(y_p-\theta)^2 |P_n\right]$ subject to the condition that expert does not want to deviate and collect $n' \neq n$ trials, when threatened with the worst possible equilibrium punishment. Evidently, the worst possible equilibrium for the expert in any communication subgame off path is the babbling equilibrium, which yields him the payoff $E\left[-\left(1/2-\theta\right)^2\right]-b^2-c\left(n'\right)$. In sum, the decision-maker will threaten the expert "not to listen to him", if he runs a number of trials other than n. This threat is credible, as babbling is an equilibrium in any communication game.[11]

As we shall see, the possibility of overinvestment extends to the case of covert information acquisition, where such an intuitive explanation is not available.

In order to formalize these results, we now solve the benchmark optimization problem of a decision maker who chooses how many trials n to perform at cost $c\left(n\right)=cn$, and then decides the action y_k^* on the basis of the signal realization $k=0,...,n$. We prove in the Appendix that

$$E\left[U^R\left(\mathbf{y}^*,\theta\right)|n\right]-c\left(n\right)=E\left[-\left(y_k^*-\theta\right)^2|n\right]-cn=-\frac{1}{6(n+2)}-cn. \tag{9}$$

Hence, the optimal number of trials $n^*\left(c\right)$ acquired by the decision maker for a given unitary cost c is such that, for generic values of c,

$$\begin{aligned} n^*\left(c\right) &= \max\left\{n:-\frac{1}{6(n+2)}-cn-\left(-\frac{1}{6(n-1+2)}-c(n-1)\right)>0\right\} \\ &= \left\lfloor\sqrt{\frac{2+3c}{12c}}-\frac{3}{2}\right\rfloor. \end{aligned} \tag{10}$$

Plugging back the expression for $n^*\left(c\right)$ in the expression for the residual variance, we obtain:

$$E\left[\left(y_{n^*}-\theta\right)^2|n^*\right]=\frac{1}{6\left(\left\lfloor\sqrt{\frac{2+3c}{12c}}-1.5\right\rfloor+2\right)}. \tag{11}$$

Next, we present three results that illustrate how an equilibrium of the overt or covert game can lead to more precise decisions than those based on information directly acquired by the decision maker.

[11] This use of babbling equilibria to improve the decision maker's welfare is reminiscent of the constructions in the sequential cheap talk models of Aumann and Hart (2003) and Krishna and Morgan (2004). Indeed, unlike in our construction, the use of babbling equilibria is there invoked also on the equilibrium path.

4.2 Overinvestment in the Overt Game

Suppose that information acquisition is overt: Deviations at the information acquisition stage are observable. If the babbling equilibrium is played off equilibrium path, overinvestment in information is possible. If it occurs, and information loss at the communication stage is not too severe, the decision taken in equilibrium is strictly more precise than when the decision maker acquires information directly.

To illustrate this possibility, we solve the simple, tractable problem to identify sufficient conditions for overinvestment in trials and full revelation of their outcome. [12]

First, we calculate the maximum number of trials $\widetilde{n}(c)$ such that, if the unitary cost is c, the utility that the expert obtains by conducting $\widetilde{n}(c)$ trials and fully revealing their realization to the decision maker is higher than the utility from running any other number of trials and playing the babbling equilibrium. That is, $\widetilde{n}(c)$ is the highest integer that satisfies $-\frac{1}{6(n+2)} - b^2 - cn \geq -\frac{1}{6(2+0)} - b^2 = -\frac{1}{12} - b^2$. Hence, the expression for $\widetilde{n}(c)$ is

$$\widetilde{n}(c) = \left\lfloor \frac{1}{12c} - 2 \right\rfloor . \tag{12}$$

Second, we recall that if (and only if) $b(n+2) \leq 1/2$, then full revelation of the outcome of the trials realizations is indeed incentive compatible. We denote by $\widehat{n}(b)$ the maximum number of trials for which the inequality is satisfied:

$$\widehat{n}(b) = \left\lfloor \frac{1}{2b} - 2 \right\rfloor . \tag{13}$$

By construction, we obtain the result that, for parameter values such that $n^*(c) + 1 \leq \max\{\widehat{n}(b), \widetilde{n}(c)\}$, the expert is willing to run at least $n^*(c) + 1$ trials in equilibrium, and to fully reveal the information to the decision maker. Simplification of this condition yields the following key possibility result. When the trial cost and the bias are not too large, the

[12] While, for simplicity, the argument hinges on the fact that noisy information is not necessarily lost in the communication between the expert and the decision maker, this feature is only sufficient, but not necessary to induce a more informed decision in the overt game than with direct information acquisition by the decision maker. What is necessary is that the information loss in the communication stage following (over)investment is not too severe.

final decision can be more precise in the overt information acquisition communication game than with direct information acquisition by the decision maker.

Proposition 2 *If* $b \leq \left(\sqrt{1+\frac{2}{3c}}+3\right)^{-1}$ *and* $c \leq \frac{5-\sqrt{17}}{48}$*, then there is at least one equilibrium of the overt game in which the final decision is more precise than the decision based on direct information acquisition by the decision maker.*[13]

The conditions in Proposition 2 are sufficient, but not necessary. To illustrate this, we numerically identified the whole region of the parameter space where the decision maker's action in her preferred equilibrium of the overt game (the one with the highest precision of the decision) is more precise than the action taken after direct information acquisition. In particular, we numerically computed the residual variance $E\left[(y_n-\theta)^2|n\right]$ in the equilibrium of the communication game with overt information acquisition preferred by the decision maker and compared it to expression (11). We ran the analysis for $b \in [0, 0.25]$ – because for $b \geq 0.25$, the unique equilibrium in the communication game is such that the expert babbles— for $c \in [0, 0.027]$ –because for $c > 0.02\overline{7}$, the unique solution of the decision maker's optimization problem is such that $n = 0$, and for $n \leq 100$ — of course, numerical analysis does not allow to consider arbitrarily large numbers of experiments.

Our results are reported in Figure 1. Figure (1a) represents the region where the sufficient conditions in Proposition 2 are satisfied. Figure (1b) reports the results of the numerical analysis.

Figure 1 confirms the insights provided by Proposition 2. When the trial cost and the bias are not too large, overinvestment more than compensates the information loss due to communication.

[13] For expositional simplicity, we have supposed that the information acquisition cost is the same for the expert and the decision maker. Proposition 2 holds a fortiori if the expert is not more efficient than the decision maker at acquiring information.

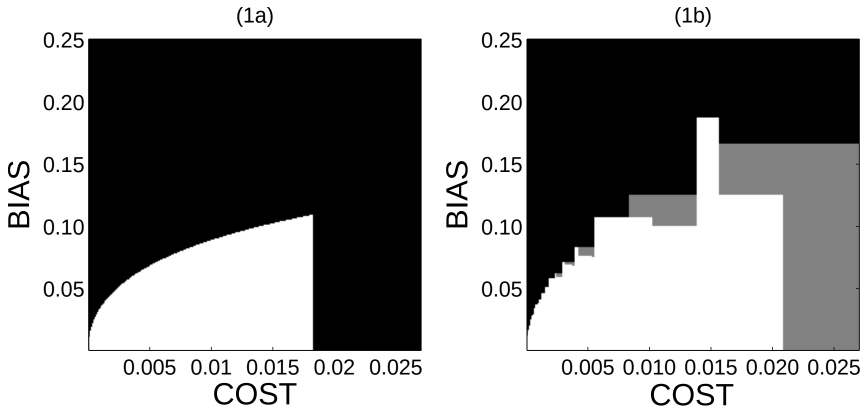

Figure 1: (1a) In the white region, the sufficient conditions in Proposition 2 are satisfied. (1b) In the white region the decision in the most informative equilibrium of the overt game is strictly more precise than with direct information acquisition. In the grey region it is as precise. In the black region it is strictly less precise.

4.3 Pareto Undominated Equilibria of the Overt Game

Our next result shows that the logic of Proposition 2 can be extended to all Pareto undominated equilibria of the overt game, to derive a weaker possibility result. We show that if $\widehat{n}(b) \geq n^*(c)$, then direct information acquisition cannot improve the decision precision over the communication game with overt information acquisition. We emphasize that this result applies to the equilibrium in which the most informative equilibrium is selected in every communication subgame, on and off the equilibrium path.

Proposition 3 *If* $b \leq \left(\sqrt{1+\frac{2}{3c}}+1\right)^{-1}$, *then direct information acquisition cannot improve the decision precision over any Pareto undominated equilibrium of the overt game.*

The condition in Proposition 2 identifies a region where the bias is not too large relative to the trial cost c. It guarantees that $\widehat{n}(b) \geq n^*(c)$, i.e. that if the expert could be induced to run $n^*(c)$ trials, he would be willing to fully reveal their realizations. The key argument in the proof shows that when this condition holds, the expert runs exactly $n^*(c)$ trials and fully reveals their outcome in his preferred equilibrium of the game. Then, by definition, the final decision in the expert's preferred equilibrium of the overt game is exactly as precise as when the decision maker directly acquires information. Because the decision maker's ex-ante utility coincides with the decision precision, it immediately follows that the decision precision in *any* Pareto undominated equilibrium of the overt game is at least as large as the precision achieved by direct acquisition.

The analysis of the expert's preferred equilibrium of the game is particularly interesting because it has the appealing feature that the most informative communication equilibrium is selected in every subgame, on and off the equilibrium path. Suppose that the condition in Proposition 2 holds, and in the expert's preferred equilibrium the expert runs $n^*(c)$ trials and fully reveals their realization. His expected equilibrium utility is $E\left[U^S\left(\mathbf{y}^*,\theta,b\right)|n\right]-c\left(n\right)$, which is the same as the payoff $E\left[U^R\left(\mathbf{y}^*,\theta\right)|n\right]-c\left(n\right)$ of a decision maker who directly conducts $n^*(c)$ trials, minus the constant b^2. What is the expert's payoff if he deviates and runs n' trials? The most informative communication equilibrium for n' trials is either fully revealing, or entails some information loss. In the former case, the expert's payoff net of b^2 would be equal to the payoff of a decision maker who directly conducted n' trials. In the latter case, it would be smaller. Because $n^*(c)$ solves the decision maker's optimization problem, also the expert has no profitable deviation to any n' different from $n^*(c)$.

4.4 Covert Game Results

Turning to the case of covert information acquisition, our results are qualitatively confirmed, although simple intuitions based on the off path play of babbling and on the information transmission loss off path are not available to support overinvestment in information acquisition.

The next Proposition shows how the logic of Proposition 3 extends to the covert game. When the bias is not too large relative to the trial cost c, direct information acquisition cannot improve the decision precision over any Pareto undominated equilibrium of the covert game.

Proposition 4 *If* $b\leq\left(2\sqrt{1+\frac{2}{3c}}+2\right)^{-1}$, *then direct information acquisition cannot improve the decision precision over any Pareto undominated equilibrium of the covert game.*

The intuition for the result is as follows: If the condition holds, then the expert runs $n^*(c)$ trials in the expert preferred equilibrium, and fully reveals the realizations to the decision maker. First, notice that if the result is true for the expert's preferred equilibrium, then it is true for any Pareto-undominated equilibrium. Second, observe that if there is an equilibrium in which n^* experiments are acquired and the outcome is fully revealed, this must be the one preferred by the expert. Next, let $\bar{n}(b)$ be the maximum number of trials for which $b(n+2) \leq 1/4$, i.e.,

$$\bar{n}(b) = \left\lfloor \frac{1}{4b} - 2 \right\rfloor. \tag{14}$$

The condition in the Proposition implies that $\bar{n}(b) \geq n^*(c)$. Because $\widehat{n}(b) \geq \bar{n}(b)$, it follows that $\widehat{n}(b) \geq n^*(c)$, and hence full revelation of the realization of $n^*(c)$ trials is incentive compatible, as long as the choice of $n^*(c)$ trials is part of the the equilibrium in the first place. The condition $\bar{n}(b) \geq n^*(c)$ guarantees that this is the case. In particular, it implies that the expert's loss from deviating from buying $n^*(c)$ trials and fully revealing the outcome is larger than the loss of a decision maker directly acquiring information, and running a number of trials $n \neq n^*(c)$. Hence, the result holds by the definition of $n^*(c)$. The proof starts from the observation that if the expert runs $n \neq n^*(c)$ trials, then he can only induce one of the equilibrium actions $y_k = E[\theta|k; n^*(c)]$ for $k = 0, ..., n^*(c)$. For some realizations of the n trials, the expert may suffer a smaller loss than the decision maker directly acquiring information. But the condition $\bar{n}(b) \geq n^*(c)$ guarantees that for any such realization of the n trials, there is another realization for which the expert loses more than the decision maker, and this larger loss more than makes up for the smaller loss in the former realization. Hence, in expectation, the expert suffers a larger loss than the decision maker directly acquiring information.

Our next result shows that also the strong result in Proposition 2 extends to the covert game: it is possible to construct Pareto undominated equilibria of the covert game where the decision is more precise than when the decision maker directly acquires information. Suppose, in fact, that the decision maker directly runs exactly n^* trials, where n^* is arbitrary. Note that this supposition implies that $[6(n^*+2)(n^*+3)]^{-1} = \underline{c}(n^*) < c < \overline{c}(n^*) =$

$[6(n^*+1)(n^*+2)]^{-1}$. Then, there exists an equilibrium where the expert at least runs n^*+1 trials and fully reveals the trial realizations to the decision maker, as long as it is the case that $\bar{n}(b) \geq n^*+1$ and that the cost c is smaller than the following threshold:

$$\hat{c}(n^*, b) = \min\left\{[6(n^*+1)(n^*+3)]^{-1}, \frac{2n^*+5}{6(n^*+3)} - b^2 + 2\sum_{j=0}^{n^*} \frac{y_j(n^*, b) - b}{n^*+1}\left(\frac{y_j(n^*, b) - b}{2} - \frac{j+1}{n^*+2}\right)\right\},$$

where

$$y_j(n^*, b) = \frac{\left\lfloor (n^*+3)\left(b + \frac{j+1}{n^*+2}\right) - \frac{1}{2}\right\rfloor + 1}{n^*+3}.$$

We present our findings as follows.

Proposition 5 *If $b \leq \left(2\sqrt{1+\frac{2}{3c}} + 6\right)^{-1}$ and $\underline{c}(n^*) < c < \hat{c}(n^*, b)$, for any arbitrary n^*, then there is at least one equilibrium of the covert game in which the decision is more precise than the decision based on direct information acquisition by the decision maker.*

The intuition for the result is as follows. Fix an arbitrary number of trials n^*, and suppose that if the decision maker directly acquires information, she optimally runs exactly n^* trials, so that $\underline{c}(n^*) < c < \overline{c}(n^*)$. The first condition in the Proposition implies that $\bar{n}(b) \geq n^*+1$. Because $\widehat{n}(b) \geq \bar{n}(b)$, this guarantees that full revelation of the realization of n^*+1 trials is incentive compatible, as long as the choice of n^*+1 trials is is part of the equilibrium in the first place. The second condition guarantees that this is indeed the case. In particular, $c > \underline{c}(n^*)$ guarantees that the expert does not deviate from n^*+1 trials to any larger trial number, with the same logic used in the Proof of Proposition 4. The condition that c is smaller than $[6(n^*+1)(n^*+3)]^{-1}$ guarantees that even a decision maker directly acquiring information would not deviate from running n^*+1 trials to run n^*-1 trials. This is true also for any number of trials smaller than n^*-1, due to the concavity of the function $E\left[-(y_k^* - \theta)^2 |n\right] = -[6(n+2)]^{-1}$. Finally, the condition that c is smaller than the second argument in the minimum defining $\hat{c}(n^*, b)$ implies that the expert does not deviate from running n^*+1 trials to run exactly n^* trials.

As in the case of Proposition 2, the conditions in Proposition 5 are sufficient, but not necessary. Again, we numerically identified the whole region of the parameter space where the decision maker's action in her preferred equilibrium of the covert game (the one with the highest precision of the decision) is strictly more precise than the action taken after direct information acquisition. Our results are reported in Figure 2. Figure (2a) represents the region where the sufficient conditions in Proposition 5 are satisfied. Figure (2b) reports the results of the numerical analysis.

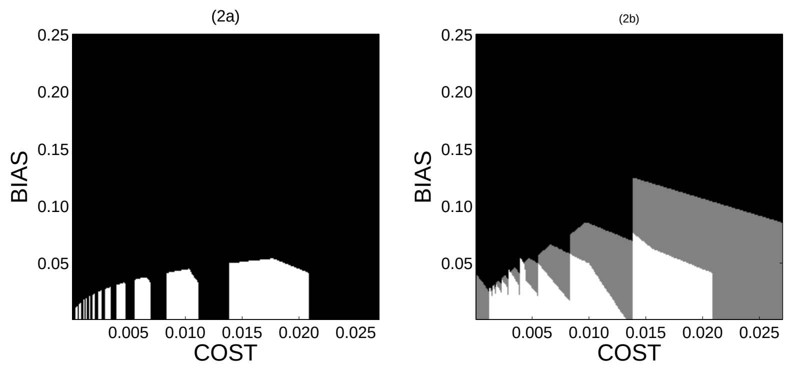

Figure 2: (2a) In the white region, the sufficient conditions in Proposition 5 are satisfied. (2b) In the white region the decision in the most informative equilibrium of the covert game is strictly more precise than with direct information acquisition. In the grey region it is as precise. In the black region it is strictly less precise.

We conclude this section by briefly discussing how our results would be affected if we considered a richer version of the covert game, in which the expert can also send a cheap talk message about the number of trials performed. In such a game, some deviations at the communication stage are observable. Suppose, without loss of generality, that the expert communicates about number of trials and trial outcome simultaneously. Suppose that the equilibrium belief is that the expert performs n trials, and reveals truthfully the number of trials run in the communication stage. Then, if the message sent by the expert is different from n, the receiver finds himself off the equilibrium path, because the equilibrium path requires that the expert sends message n. As a result, the decision maker's beliefs are free. She may believe that the number of trials is anything. Let us denote her belief by $n'(m)$. The analysis above corresponds to restricting attention to the class of equilibria with beliefs

$n'(m) = n$ for any m. These beliefs require minimal consistency in the sense that they postulate that the expert deviated only at the communication stage. Clearly, in this class of equilibria the results in Propositions 4 and 5 still hold. Consider a different set of beliefs satisfying the opposite minimal consistency: Suppose that the decision maker believes that the expert deviated at the information acquisition stage, and in particular that $n'(m) = 0$ for any $m \neq n$. For these beliefs, the decision maker will choose the action $\frac{1}{2}$ after any observable deviation. From the point of view of the expert, this set of beliefs is equivalent to the reversion to the babbling equilibrium in the overt game. Therefore, with these beliefs even the stronger result in Proposition 2 holds for the enriched covert game.

5 Organization Design

Our results have strong implications for the growing literature started by Dessein (2002), that uses communication games to explore issues of information aggregation and authority allocation in organizations. We introduce endogenous information acquisition in this literature. Specifically, we consider an organization where a principal would like to make an informed decision, and information acquisition is costly. The principal (decision-maker) may operate with an agent (expert), who has biased preferences. The principal may allocate the tasks of information acquisition and decision (authority) to the agent, or retain them for herself. Hence, there are three task allocations available to the principal:

- Centralization: The principal acquires information, paying the cost of information acquisition, and makes an informed decision
- Communication: The principal delegates the costly information acquisition task to the agent, but retains authority on the decision task
- Delegation: The principal delegates both the information acquisition and the decision task to the agent.

The trade-off between communication and delegation has been previously explored in the literature, but without information acquisition. The key result identified by Dessein (2002) and Ottaviani (2000) is that delegation outperforms communication: the principal is better off delegating the decision-making authority to the agent, than requesting his advice while retaining the decision-making authority. We now show that this result may be reversed when accounting for the fact that the level of expertise of the agent is often endogenous. Specifically, Corollary 1 below shows that communication outperforms delegation under the conditions of Propositions 3 and 4. Further, Corollary 1 shows that communication outperforms also centralization; and this is despite the assumption that the expert's cost of information acquisition is not larger than the principal's direct information acquisition cost.[14]

In order to introduce our results, we first report the principal's expected utility under centralization:

$$E\left[-\left(y_k^*-\theta\right)^2|n^*(c)\right]-cn^*(c). \tag{15}$$

and under delegation:

$$E\left[-\left(y_k^*-\theta\right)^2|n^*(c)\right]-b^2 \tag{16}$$

In both cases, the party acquiring information will run $n^*(c)$ trials as calculated in equation (10).

Turning to communication, in a given equilibrium of the overt or covert game with n trials and communication partition P_n, the principal's expected utility is:

$$E\left[-\left(y_p-\theta\right)^2|P_n\right]. \tag{17}$$

Comparing expressions (15), (16) and (17) it is immediate to see that delegation and centralization are dominated by any equilibrium of the overt or covert game for which $E\left[-\left(y_p-\theta\right)^2|P_n\right]$ is at least as large as $E\left[-\left(y_k^*-\theta\right)^2|n^*(c)\right]$. Equivalently, delegation

[14] To our knowledge, communication and centralization have not been previously explicitly compared. In fact, the previous literature typically assumed the agent is perfectly informed, whereas the principal's cost of information acquisition is infinite.

and centralization are dominated by any equilibrium of the overt or covert game such that the final decision is at least as precise as with delegation or centralization. Holding precision constant, in fact, communication is cheaper than centralization as the cost of information acquisition is off-loaded to the agent; and communication dominates delegation as it avoids making biased decisions.

Propositions 3 and 4 identify sufficient conditions under which the final decision precision $E\left[-(y_p-\theta)^2|P_n\right]$ in all Pareto undominated equilibria of the overt and covert game, respectively, is at least as large as $E\left[-(y_k^*-\theta)^2|n^*(c)\right]$. It is an immediate corollary that, under the same conditions, all Pareto undominated equilibria of the overt and covert game outperform delegation and centralization.

Corollary 1 ***(a)*** *If* $b\leq\left(\sqrt{1+\frac{2}{3c}}+1\right)^{-1}$*, then the principal strictly prefers any Pareto undominated equilibrium of the overt game to centralization and delegation.*
(b) *If* $b\leq\left(2\sqrt{1+\frac{2}{3c}}+2\right)^{-1}$*, then the principal strictly prefers any Pareto undominated equilibrium of the covert game to centralization and delegation.*

While for expositional simplicity, we have supposed that the information acquisition cost is the same for the expert and the decision maker, the comparison between delegation and the Pareto-undominated equilibria of the overt and covert game holds regardless of any information acquisition cost difference across these two subjects. The reason is that, both when communicating information to the decision maker and when delegated the final decision, it is the expert who bears the cost of information acquisition, and hence the decision maker's information cost is irrelevant in the comparison. Relaxing the assumption that the information acquisition cost is the same for the expert and for the decision maker, suppose that the cost for the expert is c^E. In the case of delegation, the expert will optimally perform $n^*\left(c^E\right)$ trials. The conditions in corollary 1, when expressed with respect to c^E rather than c, guarantee that in any Pareto-undominated equilibrium of the overt and covert game, respectively, the information transmitted is at least equal to full revelation of the outcome of $n^*\left(c^E\right)$ trials. Hence, these equilibria are preferred to the delegation outcome.

Observe that as c converges to zero, the maximum bias for which the conditions in Corollary 1 is satisfied converges to zero. Intuitively, the amount of trials performed in both delegation and centralization, $n^*(c)$, converges to infinity, therefore the highest bias for which the outcome of $n^*(c)$ trials can be credibly revealed in equilibrium converges to zero.

Corollary 1 provides sufficient conditions for communication to outperform delegation and centralization, vindicating the frequent use of this task allocation in organizations. We complete our comparison of centralization, communication and delegation by means of numerical analysis. Before presenting our results, we briefly discuss the pros and cons of the three possible task allocations.

First, we note that the principal faces a clear trade-off when choosing between delegation and centralization. By delegating, the principal off-loads the information acquisition cost to the agent, but simultaneously loses authority over the final decision. As the principal's loss from allowing the agent to take a decision increases in the agent's bias b, the principal will prefer delegation over centralization only when the bias b is small relative to the cost of information acquisition c.

Second, also the trade-off between centralization and communication is simple. Again, when the principal chooses communication over centralization, she off-loads the information acquisition cost to the agent. But, due to imperfect transmission of information, the principal bears the cost of making a less informed decision. Because the informational loss increases with the bias b, the principal prefers communication over centralization when the bias is small relative to the cost of information acquisition c.

Third, we note that the comparison between delegation and communication is more complex. In both cases, the principal's payoff decreases in the bias b, either because of the agent's biased action (under delegation), or because more information is lost in transmission with higher values of the bias (under communication). Also, in both cases the principal does not pay the cost of the information acquired.

We now present our numerical results comparing communication with delegation and centralization. Figures (3a) and (3b) respectively consider the decision maker's and the expert's preferred equilibrium of the overt game. It is remarkable that communication is the best task allocation for a large set of the cost-bias parameters. In particular, communication is optimal unless the bias is significantly large. These findings are markedly different from those in the existing literature which does not consider information acquisition and typically finds that delegation is the best task allocation.

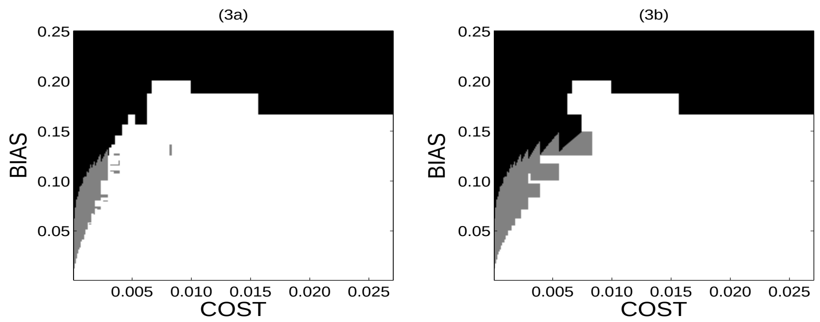

Figure 3: In the white region, the best task allocation is Communication. In the grey region, it is Delegation. In the black region, it is Centralization. Panel (3a) considers the equilibrium of the overt game preferred by the decision maker. Panel (3b) considers the one preferred by the expert.

Despite the prevalence of communication, we also observe that, for small costs of information acquisition, any of the three task allocations can be optimal, depending on the bias. Specifically, the principal prefers communication if the bias is small, delegation if the bias is intermediate, and centralization if the bias is large. But the region where delegation is optimal disappears as the information acquisition cost increases. Perhaps not surprisingly, centralization dominates when the bias is large, regardless of the cost of information acquisition. Not unexpectedly, communication is more likely to be preferred by the principal under her preferred equilibrium than under the expert's preferred equilibrium. But interestingly, this gain is mostly at the expenses of delegation, rather than centralization. Figure 4 illustrates the region of the parameter space where the sufficient condition in Corollary 1(a) is satisfied. The comparison with figure 3 shows that clearly it is only sufficient but not necessary.

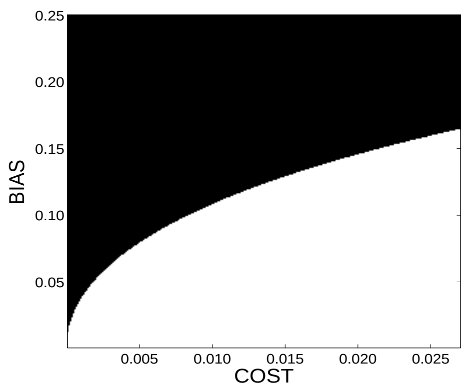

Figure 4: In the white region, the sufficient condition in Corollary 1a is satisfied.

The case of covert information acquisition is considered in Figure 5b, which compares the decision maker's preferred equilibrium to delegation and centralization. (The figure for the expert's preferred equilibrium is very similar, and hence omitted.) The qualitative features of the comparison are similar to the case of overt information acquisition. There are two main differences. First, the area where delegation is the best task allocation significantly expands; and, second, it includes also the case of large information acquisition costs. Figure 5a illustrates the region of the parameter space where the sufficient condition in Corollary 1(b) is satisfied. The comparison with figure 5b highlights that it is only sufficient but not necessary.

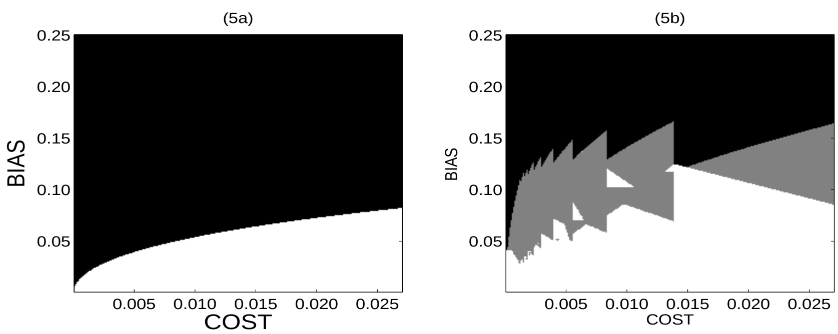

Figure 5: (5a) In the white region, the sufficient condition in Corollary 1b is satisfied. (5b) In the white region, the best task allocation is Communication. In the grey region, it is Delegation. In the black region, it is Centralization.

6 Conclusion

This paper has constructed a simple, yet rich statistical model to explore the implications of costly endogenous information acquisition in the strategic communication paradigm that was earlier developed by Crawford and Sobel (1982). One fundamental insight, if not the most important one, of previous work on strategic communication is that because information is lost in communication, the decisions based on a biased expert's advice need to be imprecise. By including costly endogenous information acquisition in the modellization, we have revisited this fundamental insight and surprisingly established that decisions based on a biased expert's advice may be more precise than optimal choices based on direct information acquisition. Strikingly, this is the case, we have shown, even if the expert is not more efficient than the decision maker at acquiring information.

As argued in the paper, this result bears important implications for organization design. A growing literature started by Ottaviani (2000) and Dessein (2002) has in fact used the strategic communication framework to give an account of authority and of the optimal authority allocation in the presence of incomplete information. Contrary to earlier results, we have found that communication by an expert to the decision maker may often outperform delegation of the decision making authority to the expert. Crucially, this result holds regardless of whether the expert is more or less efficient than the decision maker at acquiring information.

As our paper is exploratory in nature, the construction admits several directions that could lead to fruitful extension.

One natural question is for example how do overt and covert information acquisition compare from the receiver's stand point. Would a decision maker prefer knowing the amount of information acquired by an expert, although of course she cannot inspect its content? As proved by Austen-Smith (1994), indeed, the question is not transparent. His set-up, in fact, yields the result that, when the receiver is uncertain of whether the sender is informed or not, informative communication is possible for a wider range of parameter values than is possible when the receiver is sure that the sender is informed.

A second possible complement of the current analysis consists in studying the value of information in our version of the canonical example by Crawford and Sobel (1982). In fact, the work by Ivanov (2010) has in fact proved the value of information may be negative, in a statistical model where the sender may observe to which element of a partition the state of the world belongs. One natural question is whether this result would also extend to our canonical Beta-Binomial statistical model.

Appendix

Calculations leading to Expression (3). The decision maker chooses y_P so as to maximize

$$-\int_0^1 (y_p - \theta)^2 f(\theta|k \in p)\, d\theta.$$

Taking the first-order condition, we obtain $y_p = \int_0^1 \theta f(\theta|k \in p)\, d\theta = E[\theta|p]$. Simplifying:

$$E[\theta|p] = E[E[\theta|k]\,|k \in p] = \sum_{k\in p} E[\theta|k] \frac{f(k)}{\sum_{k\in p} f(k)} = \frac{1}{|p|}\sum_{k\in p}\frac{k+1}{n+2} = \frac{k+1}{n+2}$$

because $E[\theta|k] = \frac{k+1}{n+2}$, and

$$\begin{aligned} f(k) &= \int_0^1 f(k;n,\theta)\, d\theta = \frac{n!}{k!\,(n-k)!}\int_0^1 \theta^k (1-\theta)^{n-k}\, d\theta \\ &= \frac{n!}{k!\,(n-k)!}\frac{k!\,(n-k)!}{(n+1)!} = \frac{1}{n+1}. \end{aligned}$$

Proof of Proposition 1 First, we show that the incentive compatibility constraint (4) can be rewritten as

$$(y_q - y_p)\left[(y_p + y_q) - 2E[\theta/k,n] + 2b\right] \geq 0 \textit{ for all } q \in P_n.$$

The calculations are as follows:

$$\int_0^1 U^S\left(y_p\left(P_{n'}\right),\theta,b\right)f\left(\theta;k,n'\right)d\theta \geq \int_0^1 U^S(y_q\left(P_{n'}\right),\theta,b)f\left(\theta;k,n'\right)d\theta$$

$$\begin{aligned}
-\int_0^1\left[(y_p-\theta-b)^2-(y_q-\theta-b)^2\right]f(\theta;k,n)\,d\theta &\geq 0\\
-\int_0^1\left[y_p^2+(\theta+b)^2-2y_p(\theta+b)-y_q^2-(\theta+b)^2+2y_q(\theta+b)\right]f(\theta;k,n)\,d\theta &\geq 0\\
-\int_0^1\left[y_p^2-y_q^2-2(y_p-y_q)(\theta+b)\right]f(\theta;k,n)\,d\theta &\geq 0\\
-(y_p-y_q)\left[(y_p+y_q)-2E[\theta/k,n]-2b\right] &\geq 0
\end{aligned}$$

Next, we prove that in any pure-strategy equilibrium of the communication subgame, each element of the equilibrium partition is connected. Suppose by contradiction that there exists an equilibrium where at least one element of the partition is not connected. Then, there exists at least a triple of types (k,k',k'') such that $k<k''<k'$, k and k' belong to the same element of the partition, which we denote by p_a, and k'' belongs to a different element, which we denote by p_b. Let y_a and y_b be the equilibrium actions associated to p_a and p_b respectively. By incentive compatibility, the following inequalities must hold:

$$\begin{aligned}
(y_b-y_a)\left(y_a+y_b-\frac{2(k+1)}{n+2}-2b\right) &\geq 0\\
(y_b-y_a)\left(y_a+y_b-\frac{2(k'+1)}{n+2}-2b\right) &\geq 0\\
(y_a-y_b)\left(y_a+y_b-\frac{2(k''+1)}{n+2}-2b\right) &\geq 0
\end{aligned}$$

Because the first two expressions are positive, then $y_a+y_b-\frac{2(k+1)}{n+2}-2b$ and $y_a+y_b-\frac{2(k'+1)}{n+2}-2b$ have the same sign. But then, also $y_a+y_b-\frac{2(k''+1)}{n+2}-2b$ has the same sign, because $k<k''<k$. And hence, the last expression is negative: A contradiction.

Next, we prove that incentive compatibility implies expression (5). Let k be the expert's type. Denote by y the equilibrium action associated to k, and by $\widetilde{y}$ any other equilibrium action. The incentive compatibility constraint is:

$$(\widetilde{y}-y)\left(\widetilde{y}+y-\frac{2\,(k+1)}{n+2}-2b\right)\geq 0. \tag{18}$$

First, we consider the possibility that a type k deviates by inducing an equilibrium action $\widetilde{y}$ larger than y. Hence, incentive compatibility is satisfied if and only if

$$\widetilde{y}+y-\frac{2\,(k+1)}{n+2}-2b\geq 0. \tag{19}$$

Because the expression is increasing in $\widetilde{y}$ and decreasing in k, it immediately follows that the tightest incentive compatibility constraints concern the highest type k in any element p_i of the equilibrium partition, entertaining the possibility of deviating and inducing the equilibrium action $\widetilde{y}$ associated to p_{i+1}, the element of the partition immediately to the right of p.

Hence, we now consider such constraints. Letting z be the cardinality of p_i and j be the cardinality of p_{i+1}, the explicit expression for y and $\widetilde{y}$ are:

$$y=\frac{1}{z}\left[\frac{k+1}{n+2}+\frac{k-1+1}{n+2}+...+\frac{k-(z-1)+1}{n+2}\right]=\frac{2k-z+3}{2\,(n+2)}$$

$$\widetilde{y}=\frac{1}{j}\left[\frac{k+1+1}{n+2}+\frac{k+2+1}{n+2}+...+\frac{k+j+1}{n+2}\right]=\frac{2k+j+3}{2\,(n+2)}$$

Hence, condition (19) simplifies as:

$$\frac{2k+j+3}{2\,(n+2)}+\frac{2k-z+3}{2\,(n+2)}-\frac{2\,(k+1)}{n+2}-2b\geq 0,$$

or,

$$j\geq z+4b\,(n+2)-2. \tag{20}$$

Proceeding in the same fashion, we prove that when $\widetilde{y}<y$, the tightest incentive compatibility constraints concern the lowest type k in any element p_i of the equilibrium partition, entertaining the possibility of deviating and inducing the equilibrium action $\widetilde{y}$ associated

to p_{i-1}, the element of the partition immediately to the left of p_i. Again, letting j be the cardinality of p_i, and z be the cardinality of p_{i-1}, we obtain

$$y = \frac{2k+j+1}{2(n+2)} = \frac{1}{j}\left[\frac{k+1}{n+2} + \frac{k+1+1}{n+2} + ... + \frac{k+j-1+1}{n+2}\right] = \frac{2k+j+1}{2(n+2)}$$

$$\widetilde{y} = \frac{1}{z}\left[\frac{k-1+1}{n+2} + \frac{k-2+1}{n+2} + ... + \frac{k-z+1}{n+2}\right] = \frac{2k-z+1}{2(n+2)}$$

Hence, condition (19) simplifies as:

$$\frac{2k-z+1}{2(n+2)} + \frac{2k+j+1}{2(n+2)} - \frac{2(k+1)}{n+2} - 2b \leq 0$$

which implies

$$j \leq z + 4b(n+2) + 2. \tag{21}$$

Putting together the inequalities (20) and (21), we obtain condition (5). This characterization implies that a fully separating equilibrium exists if and only if $4b(n+2) - 2 \leq 0$, i.e. $b \leq \frac{1}{2(n+2)}$. Further, for $b \geq 1/4$, it follows that $4b(n+2) \geq n+2$, and hence condition (20) cannot be satisfied by any partition, other than the trivial partition $P = \{\{0, 1, ..., n+1\}\}$: The unique equilibrium is the babbling equilibrium. Finally, as $n \to \infty$, any equilibrium partition P converges to an equilibrium partition of the model by Crawford and Sobel (1982) where the expert is perfectly informed, in the sense that, for any i, $|p_i|/(n+1) \to a_i - a_{i-1}$. In fact, condition (5) implies that

$$\frac{4b(n+2)-2}{n+1} \leq \frac{|p_{i+1}| - |p_i|}{n+1} \leq \frac{4b(n+2)+2}{n+1},$$

and, taking limits for $n \to \infty$,

$$4b \leq a_i - a_{i-1} + a_{i+1} - a_i \leq 4b,$$

which is exactly the incentive compatibility condition of Crawford and Sobel (1982).

Calculations leading to Expression (6). A mean-variance decomposition yields:

$$\begin{aligned}
E\left[-\left(y_p-\theta-b\right)^2\right] &= -\int_0^1 \left(y_p-\theta-b\right)^2 d\theta \\
&= -\int_0^1 \left[\left(y_p-\theta\right)^2+b^2-2b\left(y_p-\theta\right)\right] d\theta \\
&= E\left[-\left(y_p-\theta\right)^2\right]-b^2+2bE\left[y_p-\theta\right] \\
&= E\left[-\left(y_p-\theta\right)^2\right]-b^2,
\end{aligned}$$

because $E_p\left[y_p\right]=E_p\left[E_\theta\left[\theta|p\right]\right]=E_\theta\left[\theta\right]$, by the law of iterated expectations.

Proposition 6 *For any n and b, the Pareto-efficient incentive compatible partition is $P^*=\{p_1^*,...,p_K^*\}$ such that $K=\max\{k\in\mathbb{N}|k+\lceil 4b\left(n+2\right)-2\rceil\times\frac{k(k-1)}{2})\leq n+1\}$. For all $i=1,...,K$, the element p_i^* of the equilibrium partition consists of consecutive types and has cardinality $|p_i^*|=1+\lceil 4b\left(n+2\right)-2\rceil\times\left(i-1\right)+\left\lfloor\frac{r}{K}\right\rfloor+\mathbb{I}\left\{r-\left(\left\lfloor\frac{r}{K}\right\rfloor+1\right)K+i>0\right\}$, where $r\equiv n+1-\left[K+\lceil 4b\left(n+2\right)-2\rceil\times\frac{K(K-1)}{2}\right]$, and $\mathbb{I}$ denotes the indicator function.*

Proof. The equilibrium partition P identified in the Proposition is the one with the largest cardinality K and with the smallest difference in the cardinality of subsequent elements, subject to the incentive compatibility condition (5).

First, we show that minus the expected residual variance $E_\theta\left[-\left(y_p-\theta\right)^2\right]$ can be rewritten as $-\frac{1}{3}+E\left[E(\theta|p)^2\right]$.

By the law of iterated expectations,

$$\begin{aligned}
E_\theta\left[-\left(y_p-\theta\right)^2\right] &= -E_\theta\left[\left(E\left[\theta|p\right]-\theta\right)^2\right] \\
&= -E_p\left[E_\theta\left[\left(E\left[\theta|p\right]-\theta\right)^2|p\right]\right] \\
&= -E_p\left[Var\left[\theta|p\right]\right].
\end{aligned}$$

Because $Var[\theta] = E_p[Var[\theta|p]] + Var_p[E(\theta|p)]$, we thus obtain:

$$\begin{aligned}
E_\theta\left[-(y_p-\theta)^2\right] &= -Var[\theta] + Var_p[E(\theta|p)] \\
&= -Var[\theta] + E\left[E(\theta|p)^2\right] - E[E(\theta|p)]^2 \\
&= -Var[\theta] + E\left[E(\theta|p)^2\right] - E[\theta]^2 \\
&= -\frac{1}{12} + E\left[E(\theta|p)^2\right] - \left(\frac{1}{2}\right)^2 \\
&= -\frac{1}{3} + E\left[E(\theta|p)^2\right].
\end{aligned}$$

Next, we show that among the equilibrium partitions with the largest number of elements, the equilibrium with the smallest difference between the cardinalities of any two subsequent elements is the one that maximizes $E\left[-(y_p-\theta)^2\right] = -\frac{1}{3} + E\left[E[\theta|p]^2\right]$.

Consider an equilibrium partition P with I elements $\{k_i, ..., k_{i+1}-1\}_{i=1}^I$. Denoting the associated expected residual variance by $E\left[-(y_p-\theta)^2; P\right]$ we obtain:

$$E\left[-(y_p-\theta)^2; P\right] = -\frac{1}{3} + E\left[E(\theta|p)^2\right] = -\frac{1}{3} + \sum_{i=1}^{I} \frac{k_{i+1}-k_i}{n+1}\left(\frac{k_{i+1}+k_i+1}{2(n+2)}\right)^2$$

where $k_{i+1} \equiv n+1$. Next, consider a different equilibrium partition $P' = \left\{k'_i, ..., k'_{i+1}-1\right\}_{i=1}^I$, such that there is a unique $i \in I$ with $k'_i = k_i + 1$, and $k'_j = k_j$ for all $j \neq i$. Denoting the associated expected residual variance by $E\left[-(y_p-\theta)^2; P\right]$ we obtain:

$$\begin{aligned}
& E\left[-(y_p-\theta)^2; P'\right] - E\left[-(y_p-\theta)^2; P\right] \\
= & \frac{k_{i+1}-(k_i+1)}{n+1}\left(\frac{k_{i+1}+(k_i+1)+1}{2(n+2)}\right)^2 + \frac{k_i+1-k_{i-1}}{n+1}\left(\frac{k_i+1+k_{i-1}+1}{2(n+2)}\right)^2 \\
& -\frac{k_{i+1}-k_i}{n+1}\left(\frac{k_{i+1}+k_i+1}{2(n+2)}\right)^2 - \frac{k_i-k_{i-1}}{n+1}\left(\frac{k_i+k_{i-1}+1}{2(n+2)}\right)^2 \\
= & \frac{(k_{i+1}-k_{i-1})\left[(k_{i+1}-k_i)-(k_i+1-k_{i-1})\right]}{4(n+2)^2(n+1)} > 0.
\end{aligned}$$

where the last inequality holds because P' is an equilibrium partition, hence it must be that $k'_{i+1} - k'_i = k_{i+1} - k_i - 1 > k_i + 1 - k_{i-1} = k'_i - k'_{i-1}$.

To conclude the proof, we need to show that, among the equilibrium partitions with the smallest difference in the cardinality of subsequent elements, the one which maximizes welfare is the equilibrium partition with the largest number of elements. Specifically, denoting the best equilibrium partition among those with m elements by $P(m)$, we prove that $P(j)$ dominates $P(j-1)$. Repeating the argument proves the statement.

To prove that $P(j)$ dominates $P(j-1)$ we describe an algorithm to construct a sequence of partitions with the following features:

(a) the first term of the sequence is $P(j)$

(b) the last term of the sequence is $P(j-1)$

(c) each term of the sequence, except for the last one, is a partition with j elements

(d) each term of the sequence is preferred by both players to the next one

The algorithm is the following. Given the n-th term of the sequence (the n-th partition), the $(n+1)$-th is constructed as follows:

(i) If the sub-partition that includes the largest $(j-2)$ elements of n-th partition is identical to the sub-partition that includes the largest $(j-2)$ elements of $P(j-1)$, then let the $n+1$-th partition be $P(j-1)$; i.e., let the first element of the $n+1$-th partition be equal to the union of the first *two* elements of the n-th partition. This step concludes the algorithm, and satisfies condition (d), because, for any k_1, k_2 with $k_1 > 1$, and $k_2 > k_1 + 1$,

$$\begin{aligned} & \frac{k_2 - k_1}{n+1}\left(\frac{k_2 + k_1 + 1}{2(n+2)}\right)^2 + \frac{k_1 - 1}{n+1}\left(\frac{k_1 + 1 + 1}{2(n+2)}\right)^2 - \frac{k_2 - 1}{n+1}\left(\frac{k_2 + 1 + 1}{2(n+2)}\right)^2 \\ = \; & \frac{1}{4}\frac{(k_2 - k_1)(k_2 - 1)(k_1 - 1)}{(n+2)(n+1)} > 0. \end{aligned}$$

(ii) If the sub-partition that includes the last $(j-2)$ elements of n-th partition is *not* identical to the sub-partition that includes the largest $(j-2)$ elements of $P(j-1)$, then the $(n+1)$-th partition is obtained from the n-th by moving the highest type included in the k-th element p_k^n into the $(k+1)$-th element p_{k+1}^n, where $k < j$ is the highest index that satisfies the following conditions:

(iia) for $l < j-2$, if the sub-partition that includes the last l elements of n-th partition is identical to the sub-partition that includes the last l elements of $P(j-1)$, then $k < j-l$.[15]

(iib) the cardinality of p^n_{k+1} is strictly smaller than the cardinality of the k-th element of $P(j-1)$.

(iic) if the union of p^n_1 and p^n_2 is equal to the first element of $P(j-1)$, then $k > 2$.

Because the number of types is finite, the algorithm has an end.

The type-(ii) step can be repeated exactly until the condition for the type-(i) step is satisfied because, by construction, the cardinality of the l-th element of $P(j-1)$ is weakly larger than the cardinality of the $(l+1)$-th element of $P(j)$, hence the union of the first two elements of $P(j)$ has cardinality weakly larger than the cardinality of the first element of $P(j-1)$.

Calculations leading to Expression (9). First we note that for any number of trials n and realization k, the optimal action y_k equals $E[\theta|k] = (k+1)/(n+2)$. Hence,

$$\begin{aligned}
E\left[-\left(y^*_k-\theta\right)^2|n\right]-cn &= -\sum_{k=0}^{n}\Pr(k;n)\int_0^1 \left(E[\theta|k]-\theta\right)^2 f(\theta;k,n) - cn \\
&= -\sum_{k=0}^{n}\frac{1}{n+1}\int_0^1\left(\frac{k+1}{n+2}-\theta\right)^2\frac{(n+1)!}{k!\,(n-k)!}\theta^k(1-\theta)^{n-k}\,d\theta - cn \\
&= -\sum_{k=0}^{n}\frac{1}{n+1}\int_0^1\left[\left(\frac{k+1}{n+2}\right)^2+\theta^2-2\theta\left(\frac{k+1}{n+2}\right)\right]\frac{(n+1)!}{k!\,(n-k)!}\theta^k(1-\theta)^{n-k}\,d\theta - cn \\
&= -\sum_{k=0}^{n}\frac{1}{n+1}\left[\int_0^1\theta^2\frac{(n+1)!}{k!\,(n-k)!}\theta^k(1-\theta)^{n-k}\,d\theta-\left(\frac{k+1}{n+2}\right)^2\right]-cn \\
&= -\sum_{k=0}^{n}\frac{1}{n+1}\left[\frac{(k+2)(k+1)}{(n+3)(n+2)}-\left(\frac{k+1}{n+2}\right)^2\right]-cn \\
&= -\frac{1}{6(n+2)}-cn.
\end{aligned}$$

[15] For example, if $j = 10$, if the last three elements of the $n-th$ partition in the sequence are identical to the last three elements of the target partition, then they shouldn't be changed anymore, hence $k < 7$, so that "at most" a type is taken from the 6-th element and moved into the 7-th.

Proof of Proposition 2. The condition $n^*(c) + 1 \leq \widetilde{n}(c)$ is satisfied if $\sqrt{\frac{2+3c}{12c}} - \frac{3}{2} + 1 \leq \frac{1}{12c} - 2$, i.e., $c \leq \frac{5-\sqrt{17}}{48}$, whereas the condition $n^*(c) + 1 \leq \widehat{n}(b)$ is satisfied if $\sqrt{\frac{2+3c}{12c}} - \frac{3}{2} + 1 \leq \frac{1}{2b} - 2$, or $b \leq \left(\sqrt{1 + \frac{2}{3c}} + 3\right)^{-1}$.

If $\widehat{n}(b) \geq n^*(c) + 1$ and $\widetilde{n}(c) \geq n^*(c) + 1$, then there exists an equilibrium of the overt information acquisition game in which the expert runs $n^*(c)+1$ trials and fully reveals their realizations, while the babbling equilibrium is played in any subgame in which $n' \neq n$ trials are run. Hence, the decision maker's utility $E\left[-(y_p - \theta - b)^2 | P_n\right]$ in the decision maker's preferred equilibrium must be at least $-1/[6(n^* + 1 + 2)]$ which is strictly strictly bigger than the decision maker's utility $-1/[6(n^* + 2)]$ if she had direct access to information. The proof is then concluded by noting that the final decision precision coincides with the decision maker's utility.

Proof of Proposition 3. The condition $b \leq \left(\sqrt{1 + \frac{2}{3c}} + 1\right)^{-1}$ implies that $\lfloor \frac{1}{2b} - 2 \rfloor \geq \left\lfloor \sqrt{\frac{2+3c}{12c}} - 1.5 \right\rfloor$. Consider the equilibrium in which the Pareto-efficient incentive compatible partition is played in the communication stage on and off the equilibrium path and the expert, correctly anticipating this, selects the number of trials that maximizes his expected payoff. We prove that if $\lfloor \frac{1}{2b} - 2 \rfloor \geq \left\lfloor \sqrt{\frac{2+3c}{12c}} - 1.5 \right\rfloor$, then in equilibrium the number of trials is exactly $n^*(c)$ and full revelation occurs. First, notice that $\lfloor \frac{1}{2b} - 2 \rfloor \geq \left\lfloor \sqrt{\frac{2+3c}{12c}} - 1.5 \right\rfloor$ implies that $\widehat{n}(b) \geq n^*(c)$, which in turn implies that fully revealing the outcome of $n^*(c)$ trials is incentive compatible. Next, consider deviations at the information acquisition stage. If the expert purchases less than $n^*(c)$ trials, full revelation occurs. Since $E\left[-(y_p - \theta - b)^2\right] - cn = E\left[-(y_p - \theta)^2\right] - b^2 - cn$, the difference between equilibrium payoff and deviation payoff is then equal to the payoff difference that the decision maker would receive in the single agent decision problem if he purchased less than $n^*(c)$ trials rather than $n^*(c)$. This payoff difference is negative, by definition of $n^*(c)$. If instead the expert deviates to purchasing more than $n^*(c)$ trials, full separation might or might not be incentive compatible in the subsequent communication stage. Hence, the deviation gain is weakly smaller than the payoff difference that the decision maker would receive in the single agent decision problem and again the result is implied by the definition of $n^*(c)$.

Proof of Proposition 4

The condition implies that $\bar{n}(b) \geq n^*(c)$. Because $\widehat{n}(b) \geq \bar{n}(b)$, it follows that $\widehat{n}(b) \geq n^*(c)$. Hence, full revelation is incentive compatible at the communication stage if the expert runs $n^*(c)$ trials. Further, because $n^*(c)$ is the optimal centralized number of trials, it follows that, if there is an equilibrium where $n^*(c)$ trials are run, and the realizations are fully revealed, then this equilibrium is the expert preferred equilibrium. Such equilibrium yields a higher ex-ante decision maker utility than both centralization and delegation. Because this is the expert preferred equilibrium, a fortiori, all Pareto-undominated equilibria yield a higher ex-ante decision maker utility than both centralization and delegation.

Hence, we only need to show that running $n^*(c)$ and fully revealing the realizations is an equilibrium. In order to do so, we show that for any deviation to $n \neq n^*(c)$, the expert collects a payoff $\hat{W}(n)$ —net of information acquisition costs, which is smaller than the full revelation payoff $W(n) = 1/[6(n+2)]$. Then, the definition of n^* immediately implies that $W(n^*) \geq W(n) > \hat{W}(n)$.

First, note that, for any deviation n, the expected utility of a type $j = 0, ..., n$ for inducing the outcome y_j is:

$$\begin{aligned} W(i, n; y) &= -\int_0^1 (y_j - \theta - b)^2 f(\theta|j, n)\, d\theta \\ &= -\int_0^1 \left(y_j^2 + (\theta + b)^2 - 2(\theta + b) y_j\right) f(\theta|j, n)\, d\theta \\ &= -\left[y_j^2 - 2y_j (E[\theta|j, n] + b)\right] - \int_0^1 (\theta + b)^2 f(\theta|j, n)\, d\theta, \end{aligned}$$

which is quadratic in y_j, with bliss point $E[\theta|j, n] = \frac{j+1}{n+2} + b$. The outcomes that type j may induce after the deviation are the outcomes $y_k = \frac{k+1}{n^*+2}$, for $k = 0, ..., n^* + 1$ which are compatible with the belief that n^* trials were run. Hence type j will choose the outcome $y_k(j) = \frac{k+1}{n^*+2}$ which minimizes

$$\left| \frac{k+1}{n^* + 2} - \left(\frac{j+1}{n+2} + b \right) \right|.$$

In the full revelation outcome instead, he would induce action $\frac{j+1}{n+2}$. Because the expected utility is quadratic, and hence symmetric around the bliss point, the type j is better off by inducing action $y_k(j)$ instead of action $\frac{j+1}{n+2}$ if and only if there exists $k = 0, ..., n^*$ such that:

$$\left| \frac{k+1}{n^*+2} - \left(\frac{j+1}{n+2} + b \right) \right| < b.$$

So, in the case that $b \leq 1/[4(n^*(c)+2)]$, type j is better off with $y_k(j)$ relative to $\frac{j+1}{n+2}$ if and only if there is $k = 0, ..., n^*$ such that:

$$\frac{k+1}{n^*+2} - 2b \leq \frac{j+1}{n+2} \leq \frac{k+1}{n^*+2}.$$

Taking the worse-case scenario, let's set henceforth $b = 1/[4(n^*(c)+2)]$, so that type j is better off with $y_k(j)$ relative to $\frac{j+1}{n+2}$ if and only if there is $k = 0, ..., n^*$ such that:

$$\frac{k+1}{n^*+2} - \frac{1}{2(n^*+2)} \leq \frac{j+1}{n+2} \leq \frac{k+1}{n^*+2}.$$

But note that these inequalities implies that type $n - j$ is worse off by inducing $y_k = \frac{k+1}{n^*+2}$, for any $k = 0, ..., n^*$, relative to $\frac{n-j+1}{n+2}$, because they imply

$$1 - \left(\frac{k+1}{n^*+2} - \frac{1}{2(n^*+2)} \right) \geq 1 - \frac{j+1}{n+2} \geq 1 - \frac{k+1}{n^*+2},$$

and hence

$$\frac{n^*-k+1}{n^*+2} \leq \frac{n-j+1}{n+2} \leq \frac{n^*-k+1}{n^*+2} + \frac{1}{2(n^*+2)}.$$

Under full revelation, type j's utility is:

$$
\begin{aligned}
W(j,n) &= -\int_0^1 \left(E[\theta|j,n]-\theta-b\right)^2 f(\theta|j,n) \\
&= -\int_0^1 \left[\left(E[\theta|j,n]-\theta\right)^2 - b\left(E[\theta|j,n]-\theta\right)+b^2\right] f(\theta|j,n) \\
&= -\int_0^1 \left(E[\theta|j,n]-\theta\right)^2 f(\theta|j,n) - b^2 \\
&= -\left[\left(E[\theta|j,n]\right)^2 - 2(E[\theta|j,n])\frac{j+1}{n+2}+\frac{(j+2)(j+1)}{(n+3)(n+2)}\right]-b^2 \\
&= -\left[\left(\frac{j+1}{n+2}\right)^2 - 2\left(\frac{j+1}{n+2}\right)\frac{j+1}{n+2}+\frac{(j+2)(j+1)}{(n+3)(n+2)}\right]-b^2 \\
&= -\left[\frac{(j+2)(j+1)}{(n+3)(n+2)}-\left(\frac{j+1}{n+2}\right)^2\right]-b^2
\end{aligned}
$$

If he induces action y_j, his welfare is:

$$
\begin{aligned}
\hat{W}(j,n;y_j) &= -\int_0^1 (y_j-\theta-b)^2 f(\theta|j,n)\,d\theta \\
&= -\int_0^1 \left[(y_j-b)^2+\theta^2-2\theta(y_j-b)\right]\frac{(n+1)!}{j!\,(n-j)!}\theta^j(1-\theta)^{n-j}\,d\theta \\
&= -\left[(y_j-b)^2+\int_0^1 \frac{(n+1)!}{j!\,(n-j)!}\theta^{j+2}(1-\theta)^{n-j}\,d\theta - 2(y_j-b)\int_0^1\frac{(n+1)!}{j!\,(n-j)!}\theta^{j+1}(1-\theta)^{n-j}\,d\theta\right] \\
&= -\left[(y_j-b)^2+\frac{(n+1)!}{j!\,(n-j)!}\frac{(2+j)!\,(n-j)!}{(n+3)!}-2(y_j-b)\frac{(n+1)!}{j!\,(n-j)!}\frac{(1+j)!\,(n-j)!}{(n+2)!}\right] \\
&= -\left[(y_j-b)^2-2(y_j-b)\frac{j+1}{n+2}+\frac{(j+2)(j+1)}{(n+3)(n+2)}\right].
\end{aligned}
$$

Because the types j and $n-j$ are equally likely, we only need to show that, for arbitrary j,

$$
D(j,n)\equiv W(j,n)+W(n-j,n)-\hat{W}\left(j,n;\frac{k+1}{n^*+2}\right)-\hat{W}\left(j,n;\frac{n^*-k+1}{n^*+2}\right)>0.
$$

We distinguish two cases, depending on whether $\frac{n-j+1}{n+2}+b$ is closer to $\frac{n^*-k+1}{n^*(c)+2}$ or to $\frac{n^*-k+2}{n^*(c)+2}$. In the first case, type $n-j$ chooses $y_{n^*-k}=\frac{n^*-k+1}{n^*(c)+2}$, and we calculate:

$$
D(j,n)=\frac{2\left(2k-2j+n+kn-n^*-jn^*\right)^2}{(n^*+2)^2(n+2)^2}.
$$

In the second case, type $n-j$ chooses $y_{n^*-k+1}=\frac{(n^*-k+1)+1}{n^*(c)+2}$. In this case, the expression of

$D(j,n)$ is very cumbersome. But it turns out that:

$$\frac{\partial}{\partial b} D(j,n) = -2 < 0,$$

hence we can bound $D(j,n)$ by setting b equal to its upper bound $1/\left[4(n^*(c)+2)\right]$. We obtain:

$$D(j,n) = \frac{(4k - 4j + n + 2kn - 2n^* - 2jn^* - 2)^2}{2(n+2)^2 (n^*+2)^2} > 0.$$

This concludes the proof.

Proof of Proposition 5. Fix any number of trials n^* run in centralization. The first condition implies that $b \leq 1/\left[4(n^*+3)\right]$. Hence, it implies that $b \leq 1/\left[2((n^*+1)+2)\right]$ and full revelation is incentive compatible when running n^*+1 trials.

Because $\frac{1}{6(n^*+2)(n^*+3)} = \underline{c}(n^*) < c < \overline{c}(n^*) = \frac{1}{6(n^*+1)(n^*+2)}$, and $b \leq 1/\left[4(n^*+3)\right]$ the proof of Proposition 4 —interchanging n^* with n^*+1 —- implies that deviating from n^*+1 trials to run $n > n^*+1$ trials is not profitable.

Further, $\frac{W(n^*+1)-W(n^*-1)}{2} = \frac{1}{6(n^*+1)(n^*+3)}$, and by concavity of W, $\frac{W(n^*+1)-W(n^*-k)}{k+1} > \frac{W(n^*+1)-W(n^*-1)}{2}$. Hence, requiring that $c < \frac{W(n^*+1)-W(n^*-1)}{2}$ deters all deviations from n^*+1 to n^*-k, $k = 1, ..., n^*$.

To rule out the possibility to deviate to n^* trials from n^*+1 trials, we want that $c < W(n^*+1) - \hat{W}(n^*)$. Of course, $W(n^*+1) = -\frac{1}{6((n^*+1)+2)} - b^2$. Turning to calculate $\hat{W}(n^*)$, we proceed as follows. For any $j = 0, ..., n^*$, it is easy to see that type j will send the message $\frac{k+1}{n+3}$ associated with the largest integer k such that

$$\frac{k}{n+3} + \frac{1}{2(n+3)} \leq \frac{j+1}{n+2} + b,$$

i.e.

$$k = \left\lfloor (n+3)\left(b + \frac{j+1}{n+2}\right) - \frac{1}{2} \right\rfloor.$$

This fact is immediate for $j>1$, and for $j=1$ note that

$$\begin{aligned}\left\lfloor (n+3)\left(b+\frac{1+1}{n+2}\right)-\frac{1}{2}\right\rfloor &\geq (n+3)\left(b-\frac{1}{2(n+3)}+\frac{1+1}{n+2}\right)-1\\ &= \frac{1}{2}\frac{\left(12b+n+10bn+2bn^2+6\right)}{n+2}>0.\end{aligned}$$

Hence,

$$y_j=\frac{k+1}{n+3}=\frac{\left\lfloor (n+3)\left(b+\frac{1+1}{n+2}\right)-\frac{1}{2}\right\rfloor+1}{n+3},$$

as requested by the sufficient condition.

Further, averaging across $j=0,...,n^*$, we obtain:

$$\hat{W}(n^*)=\frac{1}{n^*+1}\sum_{j=0}^{n^*}\hat{W}(j,n^*;y_j)=-\sum_{j=0}^{n^*}\frac{1}{n^*+1}(y_j-b)^2+2\sum_{j=0}^{n^*}\frac{j+1}{(n^*+1)(n^*+2)}(y_j-b)-\frac{1}{3}$$

$$=-\frac{1}{3}-\sum_{j=0}^{n^*}\frac{y_j-b}{n^*+1}\left[y_j-b-2\frac{j+1}{n^*+2}\right].$$

Hence,

$$\begin{aligned}W(n^*+1)-\hat{W}(n^*) &= -\frac{1}{6(n^*+3)}-b^2+\frac{1}{3}+\sum_{j=0}^{n^*}\frac{y_j-b}{n^*+1}\left[y_j-b-2\frac{j+1}{n^*+2}\right]\\ &= \frac{2n^*+5}{6(n^*+3)}-b^2+\sum_{j=0}^{n^*}\frac{y_j-b}{n^*+1}\left[y_j-b-2\frac{j+1}{n^*+2}\right].\end{aligned}$$

This concludes the proof of Part (b).

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