Equilibrium Non-Parametric Income Disparity

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Abstract

This paper characterizes the order statistics of employment opportunities in a general frictional job assignment model with jobs displaying lots of heterogeneity. We establish that an auction model - an environment where the workers' wages are equal to the second best available opportunity - can explain any possible distribution of wages. We also establish that a posting model - an environment where each type of firm commits ex ante to a wage policy - yields an identical distribution of wages as the auction model. Among other things, this means that the auction and posting models each imply a unique equilibrium, the marginal cost of higher job quality is positive and increasing, and the equilibrium allocation of jobs is constrained efficient. Furthermore, using the auction model, we show that the equilibrium is the solution to a simple system of independent linear equations. The only prediction that this distinguishes the auction and posting models is their implications concerning the distribution of profits among jobs of similar productivity. We derive results in both static and dynamic environments and we show how this model can incorporate aggregate shocks and worker heterogeneity. We also provide a quantitative analysis that non-parametrically fits the model to the wage distributions of different labor markets. We then draw inference about the relative efficiency of the underlying technologies in each of these markets.

1 Introduction

In many social situations, similar people face different trading opportunities (refer to Mortensen 2003). Economists ground their analysis of these stochastic social environments in two basic theories of price formation. The first theory of price formation makes the assumption that sellers commit to prices prior to their assignment to buyers. For example, in the standard model of wage dispersion, firms commit to wages prior to their random assignment to workers.(refer Burdett and Mortensen 1999). The second theory of price formation makes the assumption that sellers negotiate a price after the random assignment of sellers to buyers. For example, in the new international trade literature, innovators are randomly assigned to sectors and receive rewards equal to the difference between the value of their innovation and the next best alternative - i.e. prices are set essentially by a local auction (refer to Eaton and Kortum 2006).[1]

This paper shows that the equilibrium of price posting and auction models can be linked to the underlying order statistics associated with heterogenous jobs being randomly assigned to homogeneous workers. In the auction model, wages are determined by the second-order statistic - the second highest valuation of a workers services in their local market. And, we also establish that any second order statistic can be supported as an equilibrium to a random matching game in which firms and workers are ex ante identical. The only requirement is that the underlying technology is concave. This means that the marginal cost of job quality is positive and increasing.

[1] A liteny of other pricing structures also exists. Many of these outcomes can be expressed as the outcome of particular bargining solution. We can think of the auction and posting models as being representative of models with ex ante and ex post pricing.

One implication of this theory is that a non-parametric fit of the model to observed wage disparity between similar workers can be used to back out a unique technology for each labor market. We can then compare the technologies of each labor market and make statements about their relative efficiency.

The price posting model is also linked to order statistics. In particular, any continuous distribution of wages that is implied by the auction model is also implied by the price posting. We establish this result by first characterizing outcomes of the price posting model using methods described in Mortensen (2003) and then comparing these results to that of the auction model. We also establish that the auction and price posting models always give identical expected payoffs to workers and firms. Therefore, these models share identical predictions about the relationship between technology and equilibrium wage dispersion. However, we show that the posting and auction models differ with regards to the distribution of profits of firms as a function of their choices of technology. In particular, the price posting model allows for only two possible profit realizations while the auction model gives a continuous distribution of profit realizations, which is related to the firm type and the first-order statistic of firm productivity at each local market.

The auction and price posting models share a number of other basic properties. For example, equilibrium outcomes under both models is constrained efficient. We also establish that the computation of equilibrium outcomes is simply the solution to a system of independent linear equations. This means that there the equilibrium can always be computed quickly by simply inverting an appropriate matrix. Furthermore, the fact that the equilibrium equations are independent means that the equilibrium is always unique. Finally, we note that our model imposes no restrictions on the stochastic realization of worker productivity as function of the Poisson assignment of jobs to workers. This means that alternative approaches that impose functional forms on these stochastic processes - such as the Frechet distribution used by Eaton and Kortum (2006) - can be viewed as special cases of our general frictional assignment environment.

The quantitative section makes a non-parametric fit of the wage distribution - which is implied by the second order statistic of our frictional assignment. We then compute the corresponding first order statistics and induce the underlying technology choice set of firms. For example, our preliminary analysis reveals that the technology choice set of United States does not dominate the choice set of Denmark. In this case, lower Danish inequality is related to lower costs associated with the creation of median quality jobs than the United States.[2]

We also show how our analysis can be extended to a dynamic environment. Here, we show that the model remains tractable and that the major results of the static model carry over to the dynamic model. That is we show that the equilibrium can be characterized as the solution to linear equations. We also allow for job to job transitions and establish that the supply of new jobs relative to the number of workers in each state is a function of the aggregate state and not of the distribution of workers. This means that our framework allows for a tractable analysis of business cycle fluctuations.

The paper is organized as follows. In the next section, we describe two prototypical search environments - auctions and buyer posting. We then characterize the equilibrium of the auction model and show that this model

[2]Our technology builds in the effects of policy such as welfare benefits and progressive taxes. Julien, Kennes and King (2008) discuss how these consideration affect the equilibrium of models of this type.

can non-parametrically fit any wage distribution. We then show that a wage posting model gives identical predictions about wage disparity. We also show that wage dispersion exists because the marginal cost of higher job quality is increasing. We also establish the constrained efficiency of the decentralized equilibrium. A quantitative exercise compares the labor markets in the United States in Denmark. The following section presents the dynamic model and the final section offers some concluding remarks

2 Prototypical search environments

This section describes two prototypical economic environments with search frictions. The auction environment is a generalization of the Julien, Kennes and King (2006) model while the buyer posting environment is characterized in Mortensen (2003).

2.1 Auctions

Consider a simple auction environment with many buyers (firms) and many sellers (workers). Assume (i) the economy has a large number N of identical, risk neutral, job candidates where each candidate has one indivisible unit of labor to sell; (ii) there are $M_i = \phi_i N$ jobs of M types: $i \in \{1, 2, ..., M\}$; (iii) the productivity of a worker is y_0 if unemployed (we normalize $y_0 = 0$) and $y_i > 0$ if employed in a job of type i, where jobs are ordered by productivity with increment $\Delta > 0$ such that $y_i = i\Delta$; (iv) each job can be assigned to only one candidate; and (v) the jobs are randomly allocated to workers. The order of play is as follows. The jobs are randomly allocated to workers. Once jobs have been assigned to candidates, wages are determined through a bidding game. That is, given the number of jobs (of each type) that have approached the worker, the worker conducts an auction to determine which firm to work for.

2.2 Buyer posting

The buyer posting environment is similar to the auction environment. In particular, we maintain assumption (i) through (v). The buyer posting model differs only in how wages are determined. In particular, the order of play with buyer posting is as follows. First, each firm commits to a posted wage. The jobs are then randomly allocated to workers. Finally, once jobs have been assigned to candidates, the candidate selects the job offering the highest wage.

3 Equilibrium

This section solves the equilibrium prices and allocations of the auction and price posting environments.

3.1 Auctions

The auction model is solved using backwards induction. Once jobs have been assigned to candidates, different candidates face, in general, different ex post opportunities. Let $y(1)$ denote the best job opportunity available to worker and $y(2)$ denote their second best opportunity. All other candidates allocate their labor according to a bidding game, where each candidate sells his labor to the highest bidder, as long as the highest bid at least matches the outside option, y_0. If only one vacancy is assigned to the candidate, then the highest bid will be exactly the outside option. When at least two vacancies are assigned to the candidate,the bidding game determines that the candidate works at the vacancy with the highest valuation for the worker, and is paid the amount of the second highest valuation from among the vacancies that have been assigned to him. The wage of a worker is given by

$$w = y(2) \tag{1}$$

The revenue of the hiring firm is given by the output of the best opportunity minus the wage paid to the worker. That is

$$R(1) = y(1) - y(2)$$

The remaining firms at this worker's location produce no output and earn no revenue.

Firms are randomly assigned to workers. Therefore, the number of type i jobs is distributed Poisson with parameter ϕ_i. This means that the probability that a worker gets m_i new offers from jobs of type i is given by

$$\omega_i^m = \frac{\phi_i^m}{m!} e^{-\phi_i} \tag{2}$$

Let $\pi_i(1)$ and $\pi_i(2)$ denote the first and second order statistics. The functional forms for these statistics is given by

$$\pi_i(1) = \left(1 - \omega_i^0\right) \prod_{j=i+1}^{M} \omega_j^0 \tag{3}$$

and

$$\pi_i(2) = \left(1 - \omega_i^0 - \omega_i^1\right) \prod_{j=i+1}^{M} \omega_j^0 + \left(1 - \omega_i^0\right) \sum_{k=i+1}^{M} \left(\frac{\omega_k^1}{\omega_k^0} \prod_{j=i+1}^{M} \omega_j^0 \right). \tag{4}$$

The first order statistic is simply the probability that the worker has at least one type i job opportunity and no other job opportunities greater than i. The second order statistic is the probability that the worker has multiple type i job opportunities and no job opportunity greater than i plus the probability that the worker has at least one type i job opportunity and a single job opportunity better than the type i job. The following proposition will be used to establish general results concerning possible wage distributions.

Proposition 1 *There is a one to one mapping of $(\phi_1, ..., \phi_M)$ into all possible order statistics $\pi_i(1)$ and $\pi_i(2)$.*

Proof. An increase in ϕ_i has a negative effect on the set $\{\pi_0(1), \pi_1(1), ... \pi_{i-1}(1)\}$ a positive effect on $\pi_i(1)$ and no effect on $\{\pi_{i+1}(1), ..., \pi_M(1)\}$. >From this we can see that any order statistic is achievable by an appropriate choice of $(\phi_1, ..., \phi_M)$. Work backwards from $\pi_M(1)$ .Set ϕ_M to achieve desired value of $\pi_M(1)$, then set ϕ_{M-1} to get desired value of $\pi_{M-1}(1)$ and so on. Note that there is only route by which a particular order statistic can be created by varying $(\phi_1, ..., \phi_M)$. The same argument applies to the creation of an arbitrary second order statistic. ■

It also from the equation for wages that, in a large market, that the second order statistic can be used to characterize the wage distribution. The wage distribution of the static model, F_S, is given by the following matrix:

$$F_S = \left\{ \begin{array}{cc} 0, & \pi_0(2) \\ \Delta, & \pi_1(2) \\ 2\Delta, & \pi_2(2) \\ . & . \\ M\Delta, & \pi_M(2) \end{array} \right\} \tag{5}$$

According to proposition 1, we can establish any arbitrary vector $(\pi_0(2), \pi_1(2), ..., \pi_M(2))$.

Therefore, we can use this model to explain any possible wage distribution. In particular, let $F(w)$ be an empirical wage distribution. A simple approximation is to set

$$F(i\Delta) = F_S(i\Delta) \tag{6}$$

for all $i \in \{0, 1, ..., M\}$ and then to take the limit as M gets large such that Δ approaches zero.

The following picture illustrates this approximation under the assumption that $M = 8$.

Simulations have been done to determine the required choice of M for a good approximation. We have found that close approximation is generally achieved if $M = 30$.

3.2 Buyer posting

It is possible to derive an explicit solution to the wage posting game as follows

Proposition 2 *The distribution of wages is given by*

$$F_p(w) = \begin{cases} \frac{1}{\phi}\ln\left(\frac{y_1 - y_0}{y_1 - w}\right) & \text{for } w \in w\left(y_1\right) = [y_0, \overline{w}_1] \\ \frac{1}{\phi}\ln\left(\frac{y_2 - \overline{w}_1}{y_2 - w}\right) & \text{for } w \in w\left(y_2\right) = [\overline{w}_1, \overline{w}_2] \\ . & . \\ \frac{1}{\phi}\ln\left(\frac{y_M - \overline{w}_{M-1}}{y_2 - w}\right) & \text{for } w \in w\left(y_2\right) = [\overline{w}_{M-1}, \overline{w}_M] \end{cases} \tag{7}$$

Proof. Follows Mortensen (2003) ■

The following proposition is founded by comparing the expected payoffs of workers and firms.

Proposition 3 *The auction and buyer posting models yield identical expected payoffs for both workers and firms*

Proof. The expected payoff of a firm in the auction environment is

$$\Pi_i = \sum_{j=0}^{i} \pi_j(1)\,(y_i - y_j) \tag{8}$$

■

Let $F_p(w)$ be the cdf implied by the price posting model for a given vector $(\phi_1, ..., \phi_M)$. From () and (2) we observe that

$$F_p^{-1}(\pi_i\,(2)) \in \left(F_A^{-1}(\pi_{i-1}\,(2)), F_A^{-1}(\pi_i\,(2))\right) \tag{9}$$

This is illustrated below

The following definition is useful

Definition 4 *An environment has lots of heterogeneity if* $\phi_i > 0$ *for all* $i \in \{1, ..., M\}$

This leads to the following proposition

Proposition 5 *The auction and price posting models yield approximately identical wage dispersion if there is lots of heterogeneity.*

Proof. Proof follows from equation ■

Note that the wage posting models implies that firms receive one of two possible values of profit while the auction model yields a distribution of profit realizations. The latter distribution is a conditional distribution related to the first order statistic.

3.3 Job entry

Possible to add vacancy costs to endogenize $(\phi_1, ..., \phi_M)$. By backing out costs we could say something about how cost of high and low quality jobs compare in different countries. Which countries have favorable cost functions, etc.

Here we add an addition stage of the game. We now consider a first stage of the game, where firms decide how many vacancies, of each type, to create. We assume that each firm can create one vacancy. (Thus, we use the terms "firm" and "vacancy" interchangeably depending on the context.) The profit of a firm is equal to its output minus its vacancy creation cost and the wage it pays to the worker. The solution to the competitive economy is the solution to the following set of linear equations

$$\begin{aligned} \Pi_i &= \max\left\{\sum_{j=0}^{i-1}\pi_j\left(1\right)\left(y_i-y_j\right)-c_i,0\right\} \qquad (10) \\ 1 &= \sum_{j=0}^{M}\pi_j\left(1\right) \end{aligned}$$

The supply of vacant jobs of productivity yi is determined by free entry, so the expected profit Π_i of a vacant job of productivity y_i is equal to zero in equilibrium:$\pi_i = 0$

Proposition 6 *Any distribution of job types is feasible as a decentralized equilibrium given appropriate job creation costs..*

Proof. Let $(\pi_0(1), ..., \pi_M(1)$ with $\pi_0\left(1\right)+\pi_1\left(1\right)+...+\pi_M\left(1\right)=1$ be an arbitrary distribution first order statistic for jobs of productivity y_i at each worker. Set costs such that $V_i = 0$. To see that this is an equilibrium, consider deviations in the quantity of jobs that . If we increase ϕ_i then each of the elements from the set $\left\{\pi_0\left(1\right),\pi_1\left(1\right),...,\pi_i\left(1\right)\right\}$ all fall. ■

What restrictions does equilibrium technology dispersion place on technology. Define

Definition 7 *Technology is linear if* $y_{i+1}/c_{i+1} = y_i/c_i$ *and concave if* $y_{i+1}/c_{i+1} < y_i/c_i$

We can apply this definition to the system of equilibrium job creation conditions.

Proposition 8 *A necessary and sufficient condition for technology dispersion is 'concavity'.*

Proof. Consider the example given with 2 possible job types. The costs are computed as follows

$$\begin{aligned} C_1 &= \Delta\pi_0(1) \\ C_2 &= 2\Delta\pi_0(1) + \Delta\pi_1(1) \end{aligned}$$

Note $y_i = i\Delta$ and that C_i must increase proportionally faster than y ■

3.4 Efficiency

Possible to compare decentralized allocation to social planning problem. The social planner maximizes

$$S = \max_{\phi_0,\ldots,\phi_M} \sum_{i=0}^{M} \left(\pi_i(1)\, y_i - c_i\phi_i\right) \tag{11}$$

where $c_i = 0$. The first order conditions are equivalent to the equilibrium conditions of the decentralized economy

Proposition 9 *The social planning solution is equivalent to the decentralized economy.*

Proof. The first order condition is equivalent to (10) ■

This result is simply a generalization of Julien, Kennes and King (2007)..

4 Dynamics

Many of the main results extend to a dynamic setting. There is large number, N, of identical risk neutral workers facing an infinite horizon, perfect capital markets, and a common discount factor β. In each time period, each worker has one indivisible unit of labor to sell. Since we focus on stationary equilibria, we drop the time subscript and, whenever needed, we use an prime (') to refer to period $t+1$, two primes for $t+2$, and so on. At the start of each period there exist E_i workers in jobs of producing $y_i > 0$ where $i \in \{1, ..., M\}.$, $y_i > y_j$ for all $i > j$ and $E_0 = (N - E_1 - ... - E_M)$ unemployed workers, with production y_0. Also, at the beginning of each period, there exist $M_i^j = \phi_i^j E_j$ vacant jobs of each productivity type $i \in \{j+1, ..., M\}$ directed at workers in type $j \in \{0, ..., M\}$ jobs.[3] In each period a vacant job has a capital cost of k_i where $y_i > y_j$ for all $i > j$. Any match in any period may dissolve in at the beginning of the subsequent period with fixed probability $\rho \in (0, 1)$. In each period, any vacancy can enter negotiations with at most one worker.

Within each period, the order of play is as follows. At the beginning of the period, given the state, new vacancies enter. Next, unemployed workers and workers who are employed in low quality jobs send applications, stating their actual state of employment, to all vacancies costlessly. Once the number of entrants has been established, and job applications received, vacancies choose which workers to approach. Once new vacancies have been assigned

[3] Note that no vacant jobs of similar or lower productivity are directed at employed workers in a type i job. As should be clear, this would never be profitable since this would entail the lower productivity never being able to hire or in the case of two similar productivity jobs competing for the same worker, with Bertrand competition driving the wage up to y_i..

to candidates, wage contracts are determined through the auction mechanism. After this, production occurs and each agent receives the amount agreed to in the contract. At the end of the period, a fraction ρ of existing matches dissolves.

4.1 Wage contract determination

Let Λ_i denote the total expected discounted value of a match between a worker and a job of productivity y_i at the start of any period, with Λ_0 representing the reserve value associated with an unemployed worker. Through the auction, in a way entirely analogous to the static model, the value of a worker's wage contract Wji is equal to the expected discounted value Λj of a match between the worker and the worker's second best available job offer:

$$W_i^j = \Lambda_j$$

The per period wage is assumed to be constant over the employment duration.

4.2 Frictional job assignment

The assignment of type i jobs vacancies to workers in type j is Poisson with parameter ϕ_i^j. We denote the first and second order statistics of this assignment by $\pi_i^j(1)$ and $\pi_i^j(2)$, respectively. These are calculated using identical expressions to that used in the static model.

The number of workers in type i jobs evolves as follows.

$$H_i = \sum \pi_i^j(1) E_j$$

The fraction ρ of all jobs dissolve at the end of the period, therefore, the supply of candidates of each type $i \in \{1..., M\}$ evolves according to the following transition equations:

$$E_i' = (1-\rho)\, H_i$$

4.3 Wages

The randomness of job offers implies that the expected value of being a candidate in every period is determined by the expected wage contract one can obtain depending on their second best job opportunity. Therefore, the values of surplus are lined by which is given by

$$\Lambda_i = y_i + \beta \left(\delta_i \sum_{j=0}^{M} \Lambda_j \pi_j^0(2) + (1-\delta_i) \sum_{j=i}^{M} \Lambda_j \pi_j^i(2) \right) \tag{12}$$

Note that the value of moving to a new job is only captured by the second order statistic. In particular, the poaching firm gains the surplus in the events where the second best offer gives less surplus than the poacher The period wage of a worker in a type A job with type B second best opportunity is given by

$$\Lambda_j = w_i^j + \beta \left(\delta_i \sum_{j=0}^{M} \Lambda_j \pi_j^0(2) + (1-\delta_i) \sum_{j=i}^{M} \Lambda_j \pi_j^i(2) \right)$$

The wage of a worker in a type i job with a type j second best opportunity is given by

$$w_i^j = y_i - (\Lambda_i - \Lambda_j) \tag{13}$$

4.4 Job creation

The expected profit Π_k of a job of productivity y_k making an offer to a worker in a type i job satisfies:

$$\Pi_k^i = \max\left\{\sum_{j=i}^{j=k-1} \pi_j(1)\left(\Lambda_k - \Lambda_j\right) - C_k, 0\right\} \tag{14}$$

for jobs of type $k \in \{i+1, ...M\}$.

The supply of jobs is determined by free entry. Thus

$$\Pi_k^i = 0$$

for all $i \in \{0, ...M\}$.and $k \in \{i+1, ...M\}$.

4.5 Equilibrium

A convenient method to solve the model is to posit a grid containing M possible values of Λ. That is the surplus of a type k job is given by

$$\Lambda_i = i\Delta + \Lambda_0$$

where Λ_0 is the present value of an unemployed worker. Using () we can solve for the first order statistic associated with workers in each job type. Thus the order statistic for workers in a type i job is characterized by

$$\begin{aligned} \sum_{j=i}^{j=k-1} \pi_j(1)\left(\Lambda_k - \Lambda_j\right) &= C_k \\ \sum_{j=i}^{M} \pi_j(1) &= 1 \end{aligned}$$

which gives $M-(i-1)$ linear independent equations for the values of $\pi_j(1)$ where $j \in \{i, ..., M\}$. We can then solve for job productivity using (). That is

$$y_i = \Lambda_i - \beta \left(\delta_i \sum_{j=0}^{M} \Lambda_j \pi_j^0(2) + (1-\delta_i) \sum_{j=i}^{M} \Lambda_j \pi_j^i(2) \right)$$

Proposition 10 *The equilibrium is unique*

This follows from the fact that there () is a system of $M + 1$ linear independent equations. Therefore, for each worker in each type of job there is a unique assignment of order statistics for each possible choice of cost. Each first order statistic corresponds to a unique second order statistic. Therefore, there is a unique mapping of costs into output.

Proposition 11 *The job creation cost of higher quality jobs increases proportionally faster than the surplus generated by these jobs*

Some practical issues for numerical analysis are..

Proposition 12 *The relationship between output and costs is concave.*

The steady state number of workers in type $i \in \{0, 1..., M\}$ jobs is given by the following system of linear equations.

$$E_i = (1-\rho) \sum \pi_i^j (1) E_j$$

We can then solve for steady employment using

$$H_i = \sum \pi_i^j (1) E_j$$

4.6 Wage dispersion

The formula for the worker's wage is given by (). The worker's wage depends on their first and second best opportunity. Let π_{ij}^{AB} denote the probability that a worker in state A, B at the start of the period is matched to jobs given them a first and second best job of i, j. This probability is given by

$$\pi_{ij}^{AB} = \pi_i^A(1)\,\pi_j(2|i, A, B)$$

where $\pi_i^A(1)$ is given by () and $\pi_j(2|i, A, B)$ is given by

$$\pi_j(2|i, A, B) = \begin{cases} \pi_j^A(1) & \text{if } j > A \text{ and } j \leq i \\ 1 - \sum_{j=A+1}^{i} \pi_j^A(1) & \text{if } j = B \\ 0 & \text{otherwise} \end{cases}$$

In the steady state

$$E_{ij} = (1-\rho) \sum_{A=0}^{M} \sum_{B=0}^{M} \pi_{AB}^{ij} E_{AB} \tag{15}$$

This represents a system of $(M+1)^2$ linear equations for $(M+1)^2$ unknowns, E_{ij}.

Proposition 13 *The wage in type i jobs stochastically dominates the wage of workers in type $j < i$ jobs.*

Proof. Consider the assignment of jobs to workers that enter the period in a type i job. The second best offer of a worker obtaining a first best y_i stochastically dominates the second best offer of a worker obtains a first best offer of $y_k < y_j$. Note that the fraction of agents in state j ■

We can prove that the auction and price posting models generate equivalent present values. This matters for the characterization of costs.

4.7 Wage posting

Within each period, the order of play is as follows. At the beginning of the period, given the state, new vacancies enter. Each of these vacancies posts a wage contract, which specifies a constant wage until the worker quits. Next, unemployed workers and workers who are employed in low quality jobs send applications, stating their actual state of employment, to all vacancies costlessly. Once the number of entrants has been established, and job applications received, vacancies choose which workers to approach. Once new vacancies have been assigned to candidates, the worker accepts the best available contract. After this, production occurs and each agent receives the amount agreed to in the contract. At the end of the period, a fraction ρ of existing matches dissolves.

It is possible to derive an explicit solution to the wage posting game as follows

Proposition 14 *The distribution of wages is given by*

$$F_p(W) = \begin{cases} \frac{1}{\phi}\ln\left(\frac{\Lambda_1-\Lambda_0}{\Lambda_1-W}\right) & \text{for } w \in w\left(y_1\right) = [y_0, \overline{w}_1] \\ \frac{1}{\phi}\ln\left(\frac{\Lambda_2-\overline{w}_1}{\Lambda_2-\Lambda}\right) & \text{for } w \in w\left(y_2\right) = [\overline{w}_1, \overline{w}_2] \\ . & . \\ \frac{1}{\phi}\ln\left(\frac{\Lambda_M-\overline{W}_{M-1}}{\Lambda_{M-1}-W}\right) & \text{for } w \in w\left(y_2\right) = [\overline{w}_{M-1}, \overline{w}_M] \end{cases}$$

Proof. This is simply a dynamic discrete time extension of the static formulation given in Mortensen (2003) ■

We also can prove

Proposition 15 *The value of job surplus in the wage posting environment* *is given by*

$$\Lambda_i = y_i + \beta \left(\delta_i \sum_{j=0}^{M} \Lambda_j \pi_j^0(2) + (1-\delta_i) \sum_{j=i}^{M} \Lambda_j \pi_j^i(2) \right) \tag{16}$$

Proof. This follows from the fact that the returns to matching in the next period are

$$\sum_{j=i}^{M} \Lambda_j \pi_j^i(2)$$

which is equivalent to the auction model. ■

This leads to following proposition

Proposition 16 *The posting and auction models yield an identical job transition matrix from type $i \in \{0, 1, ..., M\}$ into type $j \in \{0, 1, ..., M\}$ jobs*

The following proposition is the analog of the static model

Proposition 17 *The posting and auction models yield identical income dispersion.*

5 Quantitative

This section makes an international comparison of labor market technologies. We first non-parametrically fit the wage distribution of each country. We then cost of quality differentiated jobs. Results will be reported in the next version of this paper, expected by September 8

6 Conclusions

It is becoming increasingly apparent that the empirical study of the labor market requires a dynamic general equilibrium model capable of explaining a multitude of facts (ref: Browning, Hansen and Heckman 1999, Sargent and Ljungqvist 2004). However, Walrasian models cannot explain unemployment because of the assumption of market clearing. And, matching models, which do explain unemployment, give only limited insights into the questions of who works with who, and who gets paid what, because the answers to these questions are largely imposed by the exogenous matching technology and sharing rule assumed. The acknowledgement of such problems has led to research in recent years to develop alternative theoretical frameworks. This paper has taken a number of steps to develop these ideas in a general framework of stochastic job matching, which is directed by the current employment opportunities of workers.

This paper characterizes the order statistics of employment opportunities in a general frictional job assignment model with jobs displaying lots of heterogeneity. We establish that an auction model - an environment where the workers' wages are equal to the second best available opportunity - can explain any possible distribution of wages. We also establish that a posting model - an environment where each type of firm commits ex ante to a wage policy - yields an identical distribution of wages as the auction model. Among other things, this means that the auction and posting models each imply a unique equilibrium, the marginal cost of higher job quality is positive and increasing, and the equilibrium allocation of jobs is constrained efficient.

We derived results in both static and dynamic environments and we show how this model can incorporate aggregate shocks and worker heterogeneity. We also provide a quantitative analysis that non-parametrically fits the model to the wage distributions of different labor markets. We then draw inference about the relative efficiency of the underlying technologies in each of these markets.

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