EQUILIBRIUM WAGE/TENURE CONTRACTS WITH ON-THE-JOB LEARNING AND SEARCH.

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Features and Contribution of the Paper.

(i) There is a large empirical literature which decomposes wages by experience (general human capital) and tenure effects (firm specific human capital). Major identification problem: experience and tenure are perfectly correlated within an employment spell. A quit, which resets tenure to zero, is not an exogenous event. [example: department salaries].

(ii) Provides a rich equilibrium theory of individual wage profiles over time. The implied distribution of wages paid has a thick right tail, quits are associated with significant wage gains, and worker wages increase on average with age, tenure and experience.

(iii) Wages are "sticky", set according to a long term employment contract. Firms construct hierarchies where promotion is by seniority. An internal promotion mechanism rewards employee loyalty which, by reducing the firm's overall quit rate, increases firm profit. Being laid-off is costly.

(iv) Extended to a directed search framework, this structure provides an alternative to the Mortensen/Pissarides/Shimer stochastic matching framework (Shi (2008), Menzio and Shi (2008) with aggregate productivity shocks and "sticky" wages.

(v) Firm fixed effects are endogenously determined. Abowd et al (1999) find firm and worker fixed effects are not correlated. Same here but there is still sorting: firms paying higher wages attract a more experienced, and thus more productive, workforce.

(vi) A major source of wage variation across all workers is that older workers are not only more experienced (greater human capital) they have found employment at better paying firms (through on-the-job search). New labour market entrants have no experience and first jobs are likely to be poorly paid. School leavers take such low paid jobs to accumulate experience while continuing to search on-the-job for better paid employment. Low paid jobs are correlated with short average tenures.

(vii) A robust response to the Hornstein et al (2008) critique of equilibrium wage dispersion models. Hornstein et al (2008) argue frictions have to be small to explain unemployment rates, but small frictions imply a too small gap between the lowest wage paid (reservation wage) and the mean wage paid. Here reservation wages are low as (i) employment is an investment in greater human capital and (ii) a foot in the door effect (entry onto the promotion ladder).

1 An Overview of Burdett and Coles (2003, Econometrica)

Time is continuous and only steady states are considered.

A unit mass of both workers and firms participate in a labor market.

Workers are homogeneous, firms are equally productive.

Constant returns technology: a firm generates revenue p per unit of time from each worker it employs.

Workers are either unemployed or employed and obtain new job offers at Poisson rate λ. Any job offer is fully described by the wage contract offered by the firm;

$w(.) \geq 0$ defined for all tenures $t \geq 0$. Job contacts are random (not directed search).

Employment legislation requires each firm treats its new hires equally (workers are equally productive).

Workers are finitely lived, die at rate $\delta > 0$.

δ also describes the flow of new (unemployed, inexperienced) workers into the market and so ensures a unit mass of workers in a steady state.

Workers cannot borrow against future earnings. As wages increase with age, worker obtains (flow) utility $u = u(w)$ by consuming current wage w. $u(.)$ strictly concave.

Unemployed workers obtain b per unit of time and $p > b > 0$.

Firms do not compete against outside offers (Postel-Vinay and Robin (2004)).

No slavery (by giving appropriate notice, workers are free to quit).

Crucial technical simplification: firms and workers have a zero rate of time preference.

The objective of any firm is to maximize steady state flow profit.

The objective of any worker is to maximize total expected lifetime utility.

1.1 Worker Optimality

An unemployed worker's expected lifetime payoff is indicated by V_u.

Let $V(t|w(.))$ denote a worker's expected lifetime payoff when employed with tenure t at a firm offering contract $w(.)$.

Let $V_0 = V(0|w(.))$ denote the value of accepting a job offer $w(.)$.

As firms may offer different contracts, let $F(V_0)$ denote the proportion of firms in the market whose job offer, if accepted, yields a worker an expected lifetime payoff no greater than V_0.

Optimal Unemployed Worker Search:

$$\delta V_u = u(b) + \lambda \int_{V_u}^{\overline{V}} [V_0 - V_u] dF(V_0). \tag{1}$$

An unemployed worker accepts a job offer if and only if it has value $V_0 \geq V_u$.

Optimal Employed Worker Search:

$$\delta V(t|w(.)) - \frac{dV(t|w(.))}{dt} = u(w(t)) + \lambda \int_{V(t|.w())}^{\overline{V}} [V_0 - V(t|w(.))] dF(V_0). \tag{2}$$

An employed worker with tenure t accepts an outside offer if and only it has value $V_0 > V(t|w(.))$. The probability a new hire will survive to be an employee with tenure t is

$$\psi(t|w) = e^{-\int_0^t [\delta + \lambda(1 - F(V(s|w(.)))] ds}. \tag{3}$$

1.2 Firm Optimality

Let $G(V)$ denote the steady-state number of workers in the economy with expected lifetime utility strictly less than V.

Suppose a firm posts $w(.)$ with corresponding value $V_0 = V(0|w)$. $\lambda G(V_0)$ then describes this firm's hiring rate. Steady state flow profit is

$$\Omega = [\lambda G(V_0)] \left[\int_0^\infty \psi(t|w)[p - w(t)]dt \right]$$

To determine the contract that maximizes Ω, B/C use a two step procedure. First, determine a firm's wage/tenure contract that maximizes its expected profit flow conditional on offering a new hire lifetime payoff V_0. Such a contract is termed an optimal contract. Assuming such an optimal contract exists, let

$w^*(t|V_0)$ denote it and let $\Pi^*(0|V_0)$ denote the firm's maximized payoff per new hire. Optimised profit can then be written as

$$\Omega^*(V_0) = \lambda G(V_0)\Pi^*(0|V_0).$$

A firm's optimization problem then reduces to choosing a starting payoff V_0 to maximize $\Omega^*(V_0)$.

Optimal Wage Tenure Contracts

A firm's optimal contracting problem is formally defined as

$$\max_{w(.)} \int_0^\infty \psi(t|w)[p - w(t)]dt \tag{4}$$

subject to (a) $w(.) \geq 0$, (b) $V(0|w(.)) = V_0$ and (c) the optimal quit strategies of workers which determine ψ (as defined by (3)).

Theorem 1

For any F satisfying A2 and any starting payoff $V_0 \in [\underline{V}, \overline{V}]$, the optimal wage-tenure contract w^* and corresponding worker and firm payoffs V^* and Π^* are solutions to the dynamical system $\{w, V, \Pi\}$ where

$$\frac{-u''(w)}{u'(w)^2}\frac{dw}{dt} = \lambda F'(V)\Pi \tag{5}$$

$$\frac{dV}{dt} = -u'(w)\frac{d\Pi}{dt} \tag{6}$$

$$[\delta + \lambda(1 - F)]\Pi - \frac{d\Pi}{dt} = [p - w], \tag{7}$$

subject to the boundary conditions:

(i) the initial condition $V(0) = V_0$, and

(ii) $\lim_{t\to\infty}\{w(t), V(t), \Pi(t)\} = (w^{\infty}, V^{\infty}, \Pi^{\infty})$, where

$$
\begin{aligned}
u(w^{\infty})/\delta &= \overline{V}, && (8)\\
V^{\infty} &= \overline{V} && (9)\\
\Pi^{\infty} &= (p - w^{\infty})/\delta. && (10)
\end{aligned}
$$

Proof - see B/C.

The optimal contract is the saddle path to this differential equation system where $(w^{\infty}, V^{\infty}, \Pi^{\infty})$ is its limiting stationary point.

w^{∞}, in equilibrium, describes the highest wage paid in the market. The contract which offers $V_0 = \overline{V}$ implies $w(.) = w^{\infty}$: perfect consumption smoothing and no quits.

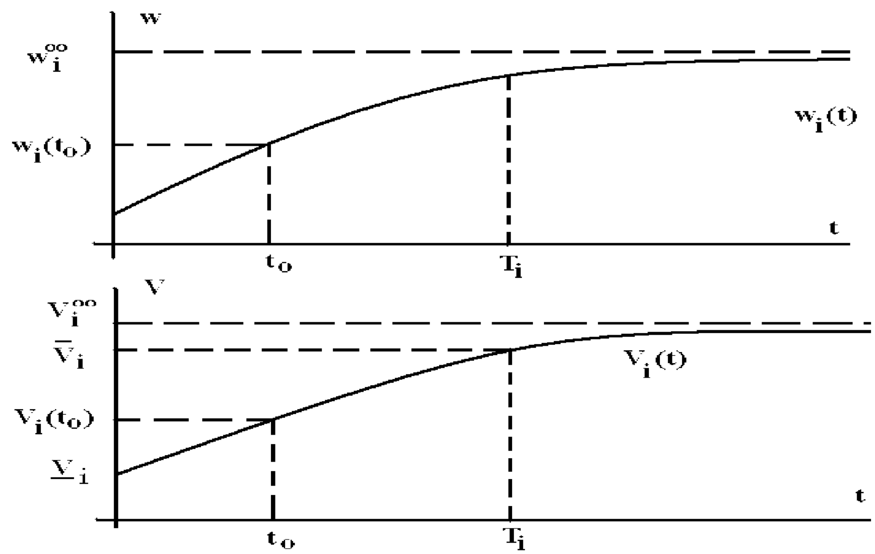

Figure 1: Type i baseline salary scale

Firms which offer $V_0 < \overline{V}$ have a positive quit rate. Figure 1 depicts the optimal contract offered by the least generous firm $V_0 = \underline{V}$. Trade-off between income smoothing and lower quit rates. Trade-off depends on profitability, poaching intensity, degree of risk aversion.

Let $w^s(t)$, $V^s(t)$, $\Pi^s(t)$ denote the optimal contract when $V_0 = \underline{V}$. Suppose a firm wishes to offer starting payoff $V_0 \in [\underline{V}, \overline{V}]$. Optimality of the baseline salary scale implies the optimal contract yielding V_0, is the starting point t_0 on the baseline salary scale where $V_i(t_0) = V_0$, and corresponding wage payments $w_i(t_0 + t)$ along the baseline salary scale at tenures $t \geq 0$.

1.3 Equilibrium Wage Tenure Contracts.

Let $G^s(t)$ denote the steady state distirbution of worker tenure points along the baseline salary scale.

$$\begin{array}{l} \Omega^*(t) = \lambda G^s(t) \Pi^s(t) = \overline{\Omega} > 0 \text{ if } dF^s(t) > 0, \\ \Omega^*(t) \leq \overline{\Omega}, \text{otherwise.} \end{array} \tag{11}$$

Problem reduces to identifying a distribution of starting point offers $F^s(.)$ which satisfies this constant profit condition, where G^S is endogenously determined by F^s and steady state turnover. Note $F(V_0)$ is determined as $F(V^s(t)) = F^s(t)$.

closed form solutions, existence, uniqueness,.....

2 Wage Tenure Contracts with On-the-job learning.

Workers are now heterogeneous - productivity $y = y_0 e^{\rho x}$ where y_0 is initial productivity, x is total labour market experience, ρ is rate of on-the-job learning. No firm specific capital.

Equal treatment requires all receive the same piece rate contract $\theta(\tau)$: a worker with productivity y and tenure τ is paid wage $w = \theta(\tau)y$.

There are job destruction shocks $\delta > 0$ (an important source of identification)

Workers now die at rate ϕ (sorry about that).

No storage - cannot use precautionary savings strategies to self-insure against layoff risk.

While unemployed, enjoy flow income by with $b < 1$.

CRRA: $u(w) = w^{1-\sigma}/(1-\sigma)$ with $\sigma > 0$.

Otherwise identical structure.

2.1 Worker Optimality

Let $V = V(y, \tau|\theta)$ denote the expected lifetime value of a worker with current productivity y and tenure τ on piece rate contract $\theta(\cdot)$. CRRA implies a separable solution of the form

$$V = y^{1-\sigma} U(\tau|\theta),$$

where $U(\tau|\theta)$ is defined as the piece rate value of the contract (with tenure τ). Let $U_0 = U(0|\widetilde{\theta})$ denote the piece rate value of the outside offer $\widetilde{\theta}$.

Let $V^U(y)$ denote the expected lifetime value of an unemployed worker with productivity y and the following identifies a separable solution of the form

$$V^U = y^{1-\sigma} U^U,$$

where U^U is a constant to be determined.

As firms offer different contracts, let $F(U_0)$ denote the proportion of firms in the market whose job offer, if accepted, yields piece rate value no greater than U_0. Search is random.

Optimal Unemployed Worker Search:

$$\phi V^U = u(by) + \lambda \int_{\underline{U}}^{\overline{U}} \mathsf{max}[y^{1-\sigma} U_0 - V^U, 0] dF(U_0),$$

where $V^U = y^{1-\sigma} U^U$. CRRA ensures all the y terms cancel out, thus

$$\phi U^U = \frac{b^{1-\sigma}}{1-\sigma} + \lambda \int_{U^U}^{\overline{U}} [U_0 - U^U] dF(U_0). \tag{12}$$

Thus each unemployed worker accepts any job offer with piece rate value $U_0 \geq U^U$; i.e. all use the same reservation piece rate value (phew).

Optimal Employed Worker Search

$$\phi V(y,\tau|\theta(.)) = \left[\begin{array}{c} u(\theta(\tau)y) + \frac{\partial V}{\partial y}\rho y + \frac{\partial V}{\partial \tau} + \delta[V^U - V] \\ +\lambda \int_{\underline{U}}^{\overline{U}} \max[y^{1-\sigma}U_0 - V, 0]dF(U_0) \end{array} \right], \tag{13}$$

where $V(.) = y^{1-\sigma}U(\tau|\theta)$. CRRA implies all the y terms all cancel out and $U(\tau|\theta)$ is given by

$$[\delta+\phi-\rho(1-\sigma)]U - \frac{dU}{d\tau} = \frac{[\theta(\tau)]^{1-\sigma}}{1-\sigma} + \delta U^U + \lambda \int_U^{\overline{U}} [1-F(U_0)]dU_0 \tag{14}$$

Again this preference structure ensures each worker's optimal quit strategy is independent of productivity y : the worker quits to any outside offer with value $U_0 \geq U = U(\tau|\theta)$. The probability a new hire survives to be an employee with tenure τ is then

$$\psi(\tau|\theta) = e^{-\int_0^\tau [\phi+\delta+\lambda(1-F(U(s|\theta)))]ds}. \tag{15}$$

2.2 Firm Optimality.

Let $\overline{ue}$ denote the steady state unemployment rate

Let $N(x)$ denote the fraction of unemployed workers who have experience no greater than x.

Let $H(x, U)$ denote the proportion of employed workers who have experience no greater than x and piece rate value no greater than U. Each of these objects are determined endogenously.

Consider now a firm which posts contract $\theta(.)$ with starting piece rate value $U_0 = U(0|\theta) \geq U^U$. The firm's steady state flow profit can be written as

$$\Omega(\theta) = \lambda \left[\begin{array}{c} \overline{ue} \int_{x=0}^{\infty} [\int_0^{\infty} \psi(\tau|\theta))[1-\theta(\tau)][y_0 e^{\rho x}] e^{\rho \tau} d\tau] \, dN(x) \\ +(1-\overline{ue}) \int_{U'=\underline{U}}^{U_0} \int_{x=0}^{\infty} [\int_0^{\infty} \psi(\tau|\theta)[1-\theta(\tau)][y_0 e^{\rho x}] e^{\rho \tau} d\tau] \, dH(x, U') \end{array} \right]$$

This condition can be re-expressed as

$$
\begin{aligned}
\Omega(\theta) &= \lambda y_0 \left[\int_0^{\infty} \psi(t|\theta)[1-\theta(t)]e^{\rho t}dt\right] \\
&\quad \times \left[\overline{ue}\int_{x=0}^{\infty} e^{\rho x}dN(x) + (1-\overline{ue})\int_{U'=\underline{U}}^{U_0}\int_{x=0}^{\infty} e^{\rho x}dH(x,U')\right].
\end{aligned}
$$

To determine the contract that maximizes Ω, I use the same two step procedure

(i) identify the optimal piece rate contract $\theta^*(.)$ which maximizes

$$\left[\int_0^{\infty} \psi(t|\theta(.))[1-\theta(t)]e^{\rho t}dt\right],$$

conditional on the contract yielding piece rate value U_0. Let

$$\Pi^*(0|U_0) = \int_0^{\infty} \psi(t|\theta^*)[1-\theta^*(t|U_0)]e^{\rho t}dt.$$

An optimal contract thus yields steady-state flow profits

$$\Omega^*(U_0) = \lambda y_0 \Pi^*(0|U_0) \left[\begin{array}{c} \overline{ue} \int_{x=0}^{\infty} e^{\rho x} dN(x) \\ +(1 - \overline{ue}) \int_{U'=\underline{U}}^{U_0} \int_{x=0}^{\infty} e^{\rho x} dH(x, U') \end{array} \right].$$

The firm's optimization problem then reduces to choosing a starting payoff U_0 to maximize $\Omega^*(U_0)$.

Optimal Piece Rate Contract solves

$$\max_{\theta(.)} \int_0^\infty \psi(t|\theta)e^{\rho t}[1-\theta(t)]dt \tag{16}$$

subject to (a) $\theta(.) \geq 0$, (b) $U(0|\theta(.)) = U_0$ and (c) the optimal quit strategies of workers which determine the survival probability $\psi(.|\theta)$.

Theorem 1

For any $U_0 \in [\underline{U}, \overline{U}]$, an optimal contract $\theta^*(.|U_0)$ and corresponding worker and firm payoffs U^* and Π^* are solutions to the dynamical system $\{\theta, U, \Pi\}$ where

$$\dot{\theta} = \frac{\lambda \left[\theta^{1-\sigma}\right]}{\sigma} F'(U)\Pi - \rho\theta \tag{17}$$

$$\dot{\Pi} = [\delta + \phi - \rho + \lambda(1 - F(U))]\Pi - (1 - \theta) \tag{18}$$

$$\dot{U} = -\theta^{-\sigma}\dot{\Pi} \tag{19}$$

with initial value $U(0) = U_0$.

(17) implies the wage paid along the optimal contract evolves according to

$$\frac{\frac{d}{dt}[y_0 e^{\rho x_0} e^{\rho t}\theta(t)]}{y_0 e^{\rho x_0} e^{\rho t}\theta(t)} = \frac{\lambda F'(U)\Pi}{\sigma\theta^{\sigma}} > 0$$

as in B/C. In B/C with no learning by doing, the most generous contract offered in the market, $U_0 = \overline{U}$, implies a constant wage (perfect consumption smoothing) and the worker never quits to a competing firm. Here instead a constant wage (perfect consumption smoothing) requires a piece rate $\theta(\tau)$ which declines at rate ρ. Thus even though an optimal contract implies wages must always increase within an employment spell, tenure effects may now be negative; i.e. $\theta^*(.)$ might be a decreasing function. As a highly paid new entrant is not at risk of being poached, he/she finds his/her wage only slowly increases over time, falling behind his/her improving productivity through on-the-job learning.

Formally the optimal contract is a saddle path solution to the differential equation system (17)-(19) with stationary point

$$[\theta^\infty]^\sigma = \frac{\lambda}{\rho\sigma} F'(U^\infty)\Pi^\infty \tag{20}$$

$$\Pi^\infty = \frac{1-\theta^\infty}{\delta + \phi - \rho + \lambda(1 - F(U^\infty))}. \tag{21}$$

$$[\delta + \phi - \rho(1-\sigma)]U^\infty = \frac{[\theta^\infty]^{1-\sigma}}{1-\sigma} + \delta U^U + \lambda \int_{U^\infty}^{\overline{U}} [1 - F(U_0)]dU_0 \tag{22}$$

There are two types of optimal contracts, initially generous ones whose value converges to U^∞ from above, and initially ungenerous ones whose value converge to U^∞ from below. Figure 1 depicts the corresponding contracts $\theta^*(.)$ in tenure space.

Figure 1 here.

As before, define *baseline piece rate scales etc.*

2.3 MARKET EQUILIBRIUM

A moment's reflection establishes that new hires do not care about the particular tenure contract that is offered, only the value U_0 obtained by accepting it. To proceed I transform the equations obtained above into value space (U).

Recall that for any starting value $U_0 \in [\underline{U}, \overline{U}]$, I can identify a starting point on the piece rate salary scales where the optimal contract yields starting payoff $U^s(t_0) = U_0$, yields maximised profit $\Pi^s(t_0)$ and identifies the corresponding piece rate paid θ. I can thus define $\theta = \widehat{\theta}(U_0) \equiv [U^s]^{-1}(U_0)$ as the piece rate paid when the worker enjoys U_0 on the baseline piece rate scales, and $\Pi = \widehat{\Pi}(U_0)$ as the firm's contination profit.

Claim 1: For $U \in [\underline{U}, \overline{U}]$, $\widehat{\Pi}$ evolves according to the differential equation

$$\frac{d\widehat{\Pi}}{dU} = -\widehat{\theta}^{\sigma} \tag{23}$$

subject to the boundary condition $\widehat{\Pi} = \Pi^{\infty}$ at $U = U^{\infty}$.

By construction, each firm's optimized steady state flow profit by offering $U_0 \in [\underline{U}, \overline{U}]$ is

$$\Omega^*(U_0) = \lambda \widehat{\Pi}(U_0) \left[\begin{array}{c} \overline{ue} \int_{x=0}^{\infty} y_0 e^{\rho x} dN(x) \\ +(1 - \overline{ue}) \int_{U'=\underline{U}}^{U_0} \int_{x=0}^{\infty} y_0 e^{\rho x} dH(x, U') \end{array} \right]. \tag{24}$$

I now formally define a Market Equilibrium.

A **Market Equilibrium** is a distribution of optimal contract offers, with corresponding value distribution $F(U)$, such that optimal job search by workers and steady state turnover implies the constant profit condition:

$$\begin{aligned} &\Omega^*(U_0) = \overline{\Omega} > 0 \text{ if } dF(U_0) > 0, \\ &\Omega^*(U_0) \leq \overline{\Omega}, \text{otherwise,} \end{aligned} \tag{25}$$

Identifying a Market Equilibrium simply requires finding $F(.)$ so that the above constant profit condition is satisfied. I perform this task using a series of lemmas .... and a miracle occurs - the characterisation of a Market Equilibrium is remarkably straightforward. Indeed, it does not depend on y_0 and so equilibrium generalises trivially to ex-ante heterogeneous workers y_0 with ability distribution A. Numerical example still to do.

3 Conclusion.

Equilibrium wage earned by worker i employed at firm j with experience $x = x_{it}$ and tenure $\tau = \tau_{it}$ at date t can be decomposed as:

$$\log w_{ij}(x, \tau) = \log y_i + \log \theta_j(0) + \rho x + \log \frac{\theta_j(\tau)}{\theta_j(0)},$$

where ρ is the rate of human capital accumulation while employed.

Note worker fixed effect $\log y_i$ is orthogonal to the firm fixed effect $\log \theta_j(0)$ [e.g. Abowd et al (1999)]

Firm fixed effects are endogenously determined.

Firm tenure effects are not orthogonal to the firm fixed effect (see figure 1).

A standard regression equation of the form

$$\log w_{ij}(x,\tau) = \log y_i + \log \theta_j(0) + \rho x + g\tau$$

is flawed as it assumes g does not vary systematically across firms.

In the equilibrium identified, the least generous firm j, the one which offers the lowest value contract in the market, offers a contract with the strongest positive tenure effect. In contrast the most generous firm j, the one which offers the highest value contract in the market, offers a contract which has a negative tenure effect. A highly paid new entrant is not at risk of being poached. He/she finds his/her wage only slowly increases over time, falling behind his/her improving productivity through on-the-job learning. The estimated "average" tenure effect, g, may be small even though (firm specific) wage tenure effects are large.

Note job destruction shocks identify experience effects. Dustmann and Meghir (2005) then find tenure effects are large and siginificant only at relatively short tenures and not for all firms.